

Basics of the course on Complex Spaces

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What is the definition of the ring of power series (formal and convergent)?.

Explain the inclusion $\mathcal{O}_{n-1}[z_n] \subset \mathcal{O}_n$.

What is a z_n -general power series?

What is a Weierstrass polynomial?

Formulate the preparation theorem and the division theorem!

Explain the isomorphism

$$\mathcal{O}_{n-1}[z_n]/Q\mathcal{O}_{n-1}[z_n] \cong \mathcal{O}_n/Q\mathcal{O}_n$$

for Weierstrass polynomials!

What are the basic algebraic properties of \mathcal{O}_n ? (\mathcal{O}_n is ZPE, \mathcal{O}_n is noetherian.)

What is an analytic algebra?

What means noether normalization for an analytic algebra?

What is the dimension of an analytic algebra?

What is a geometric realization of an ideal $\mathfrak{a} \subset \mathfrak{m}_n$?

Formulate the Hilbert–Rückert nullstellensatz!

What is a presheaf?

What is a sheaf?

What are germs and stalks?

Describe the category of presheaves and sheaves!

What is an exact sequence of presheaves?

What is an exact sequence of sheaves?

Describe the functor “associated sheaf”!

Describe the category of modules over a sheaf of rings!

Describe the canonical flabby resolution!

How are the cohomology groups of sheaves defined?

What is the long exact sequence in sheaf cohomology?

What are acyclic sheaves?

What means paracompact?

Explain (roughly) the acyclicity of sheaves of continuous or differentiable functions (and modules over them)!

What is Čech cohomology?

Describe a link between Čech and usual cohomology!

What states the lemma of Poincarè?

What states the Lemma of Dolbeault?

Explain $H^q(D, \mathbb{C}_D) = 0$, $q > 1$, for convex D !

Explain $H^q(Q, \mathcal{O}_Q) = 0$, $q > 1$, for polydiscs!

What is a coherent sheaf of rings?

What states Oka's coherence theorem?

What states Cartan's coherence theorem?

What is a coherent module over a coherent sheaf of rings?

What are the basic permanence properties of coherent modules?

What is a Stein space?

Give examples of Stein spaces!

What states the Cartan gluing lemma?

What states the (analytic) syzygy theorem

What means the exhaustion of Stein spaces by Oka domains?

What state Theorem A and B for Stein spaces?

Explain the Cousin problems and answers to them.