

Sumerian Traces in Archmimedes' Cattle Problem

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1 Introduction

The question whether Greek mathematics was influenced by Babylonian culture is still debated by historians, even though some Greek authors repeatedly pointed out that some of their oldest mathematicians, in particular Thales of Milet and Pythagoras of Samos, had travelled to Egypt and Mesopotamia. In addition we have the testimony of Josephus [8] that Abraham brought mathematics from his hometown Ur in Southern Mesopotamia to Egypt:

He communicated to them Arithmetick; and delivered to them the science of Astronomy. For before Abram came into Egypt they were unacquainted with those parts of learning: for that science came from the Chaldeans into Egypt; and from thence to the Greeks also.

Perhaps we should not take this at face value; a reasonable assumption might be that Josephus had learned about the transmission of mathematics and astronomy from Babylonia to Egypt and Greece for example in the history of Babylonia written by Berossos, which is lost except for a few fragments. Berossos lived in the times of Alexander the Great and, according to Vitruvius, later moved to the Greek island of Cos to open a school of astronomy (see [5]).

In this article we will add another example supporting a Babylonian influence on Greek mathematics to the material collected in particular by Jöran Friberg [2, 3] and Jens Høystrup [6, 7].

2 The Cattle Problem of Archimedes

As is well known, in 1773 Gotthold Ephraim Lessing discovered a manuscript in the library of Wolffenbüttel that claimed to be authored by Archimedes. We will be content with quoting just a few lines (for the complete text, see the Archimedes web page [10] maintained by Chris Rorres):

*Understand, stranger, that the white bulls
were equal to a half and a third of the black
together with the whole of the yellow,
while the black were equal
to the fourth part of the dappled and a fifth,
together with, once more, the whole of the yellow.*

Chris Rorres also points out that Homer's *Odyssey* contains a few lines reminiscent of this problem:

*Thou shalt ascend the isle triangular,
Where many oxen of the Sun are fed,
And fatted flocks. Of oxen fifty head
In every herd feed, and their herds are seven;
And of his fat flocks is their number even.*

The “triangular” island is Sicily, which is also mentioned in Archimedes' cattle problem; of course Archimedes lived (and died) in Syracuse, a city in Sicily. Thus already in the times of Homer, long before Thales and Pythagoras imported Egyptian and Mesopotamian mathematical ideas to the Ionian Islands and Southern Italy, a few verses of a song on the cattle of the sun god Helios were being passed around in Greece.

The first part of the cattle problem demands finding a solution to the following system of equations, where W , B , D and Y denote the number of white, black, dappled and yellow bulls, respectively:

$$W = \left(\frac{1}{2} + \frac{1}{3}\right)B + Y, \quad B = \left(\frac{1}{4} + \frac{1}{5}\right)D + Y, \quad D = \left(\frac{1}{6} + \frac{1}{7}\right)W + Y.$$

The second part introduces more unknowns w , b , d and y , the numbers of white, black, dappled and yellow cows, and more equations, all of them looking similar to those given above. The actual solution of the full problem was way beyond the powers of even Archimedes (see Lenstra [9]).

3 Babylonian Influence

Black et al. [1] have collected examples of Sumerian literature. One of them is a song (see [4]) dedicated to the moon god Nanna and his herds, which represent the stars (an image to which we owe the “milky way”). On p. 146, the following lines of this song are given:

*The cows are driven together in herds for him.
His various types of cow number 39 600.
His young cows and calves number 108 000.
His young bulls number 126 000.
The sparkling-eyed cows number 50 400.
The white cows number 126,000.
The cows for the evening meal
are in four groups of five each.
Such are the various types of cow of Father Nanna.
His wild cows number 180,000.
The . . . cows are four.
Their herds of cattle are seven.
Their . . . herdsmen are seven.*

It is remarkable that traces of this song can be found more than 1000 years later in Homer’s *Odyssey*.

The numbers in this Sumerian song are, as is observed in [1], multiples of 3600. In fact there’s more to them, as the following factorizations show:

$$\begin{aligned} 180\,000 &= \left(\frac{1}{2} + \frac{1}{3}\right) \cdot 60^3, \\ 126\,000 &= \left(\frac{1}{3} + \frac{1}{4}\right) \cdot 60^3, \\ 108\,000 &= \frac{1}{2} \cdot 60^3, \\ 50\,400 &= \frac{1}{3} \left(\frac{1}{2} + \frac{1}{5}\right) \cdot 60^3, \\ 39\,600 &= \frac{1}{2} \left(\frac{1}{5} + \frac{1}{6}\right) \cdot 60^3. \end{aligned}$$

These numbers reveal an affinity to the equations given in Archimedes’ cattle problem which is not easily dismissed as being accidental. Was Archimedes inspired by a Babylonian version of the song of the herds of Nanna when he composed his problem?

The fact that the cattle problem seems to be connected to the song about the herds of Nanna also seems to show that the manuscript on the cattle of the sun god is not a forgery produced by some mathematician in the middle

ages: the problem must have been composed at a time when Babylonian myths were still around.

References

- [1] J. Black, G. Cunningham, E. Robson, G. Zólyomi, *The literature of ancient Sumer*, Oxford Univ. Press 2004
- [2] J. Friberg, *Unexpected Links between Egyptian and Babylonian Mathematics*, World Scientific 2005
- [3] J. Friberg, *Amazing Traces of a Babylonian Origin in Greek Mathematics*, World Scientific 2007
- [4] M.G. Hall, *A hymn to the moon-god, Nanna*, *Journal of Cuneiform Studies* **38** (1986), 152–166; <http://www.jstor.org/stable/1359797>
- [5] J. Haubold, *The world of Berossos: Introduction*, in *he world of Berossos*, J. Haubold, G.B. Lanfranchi, R. Rollinger, J. Steele (eds.), Wiesbaden 2013
- [6] J. Høyrup, *Mesopotamian Calculation: Background and Contrast to Greek Mathematics*, IX Congresso della Società Italiana di Storia della Matematica, 2011
- [7] J. Høyrup, *Les lais: or, what ever became of Mesopotamian Mathematics?*, in: *From the banks of the Euphrates. Studies in honor of Alice Louise Slotsky* (M. Ross, ed.); 2008, 99–119
- [8] Josephus, *Antiquities of the Jews, Book I*, <http://sacred-texts.com/jud/josephus/ant-1.htm>
- [9] H. Lenstra, *Solving the Pell equation*, *Notices of the AMS* **49** (2002), 182–192; <http://www.ams.org/notices/200202/fea-lenstra.pdf>
- [10] C. Rorres, *Archimedes*, <https://www.math.nyu.edu/~crrorres/Archimedes/contents.html>

4 P.S.

The referee of HM did not think very highly of this contribution:

There is no reason to publish the item. Of course it is true that the “Archimedes” problem refers to the Homeric passage, but that was already pointed out by Heiberg in his edition (vol. II, p. 450, n. 1) – Lessing has probably taken it for granted that the reader would recognize Homeric reference of the “cattle of Helios”.

All that is shown by the Sumerian hymn is that immense riches in cattle were also interesting in Bronze Age Mesopotamia; so they probably were with all herding populations, of which there were many around the eastern Mediterranean. That the Archimedean “problem must have been composed at a time when Babylonian myths were still around” only follows for those who believe that all communication in Antiquity took place via books that are still known to us.

The reference system betrays that the author is mathematician. He should be taught that in articles about the history of mathematics, page references are given, not only books, and that web references should indicate the date of access – reference no. 10 is already outdated.

I thank him or her for pointing out Heiberg’s comment concerning Homer. Let me say that I am not in any way disappointed about the rejection of this manuscript – I was more or less expecting it. I am a bit perplexed, however, by the remark concerning the herding populations in the Eastern Mediterranean. Is there a connection between them and the topic of this article, in particular the stars in the sky? And what about sheep, goats and pigs?

I also cannot see how the remark that “the problem must have been composed at a time when Babylonian myths were still around” implies that I think that Archimedes must have consulted a book, or that all communication in Antiquity took place via books that are still known to us. I was half expecting a comment to the effect that I must have been thinking these books were written in cuneiform script.

The main point of my contribution was, in my opinion, pointing out that the numbers showing up in the song about the herds of Nanna have

a structure that is similar to the equations in the Cattle Problem. I feel flattered by the fact that the referee didn't make any derogatory comments about it. Actually he didn't comment it at all.

Finally I also plead not guilty concerning my not giving page numbers. I did give the page number for [1], and quoted the whole books by Friberg as sources showing a Babylonian influence on other cultures. All the other references are articles, with the exception of Josephus' book, which is available online and can be searched easily. Oh, and I resent the remark that I "should be taught" something. I still am learning a lot these days, but I'm past the age where I like being told that I should be taught something.

To the referee,
with the Swabian Greeting that sounds just as nice in Irish Gaelic.

P.P.S.

For more connections between Homer's Ilias and Odyssee one the one hand and Babylonian myths on the other see Walter Burkert, *Die Griechen und der Orient*, C.H. Beck 2004. And no, no page numbers: This is an excellent book that should be read from cover to cover.