

The Time Arrow in Quantum Cosmology

Our mistake is not that we take our theories too seriously, but that we do not take them seriously enough. (Stephen Weinberg 1977)
– Well, but *which* theories?

The founders of quantum theory invented their theory as a theory of atoms, that was soon successfully applied also to other microscopic systems. Macroscopic objects were thought to require the established classical concepts even though they consist of atoms. This hardly consistent traditional point of view (that would also exclude quantum cosmology) seems to be slowly changing under the impact of more recent interpretations, which allow one to describe the world in terms of a universally valid quantum theory (Sect. 4.6).

Another obstacle to quantum cosmology is that a description of the whole Universe seems to require a ‘theory of everything’, which is elusive. While there are various mathematically deep and physically even plausible proposals for such a theory, physics is an empirical science. Physical cosmology should therefore only extrapolate empirically founded concepts and laws. Mathematical cosmological models may be important and interesting in their own right, and some of them may prove physically successful in the future, but reality has usually offered great conceptual surprises that could not have been foreseen by mathematical reasoning or pure logic.

Physical cosmology should not therefore rely on any details of unconfirmed unified quantum field theories, for example. Only the general framework of quantum theory may be regarded as empirically sufficiently founded to draw cosmological conclusions from it. This framework includes, first of all, the superposition principle and the unitarity of dynamics (in other words, a general wave function and a Schrödinger equation). In cosmology, this requires an answer to the fundamental problem of what quantum theory *means* in the absence of external observers or measurement devices. Physical cosmology must therefore depend on the interpretation of quantum theory (as discussed in Sect. 4.6) in an essential way. A pragmatic probability interpre-

tation with respect to external observers is obviously ruled out, since the very concept of cosmology presumes an objective (though in principle hypothetical) reality. Quantum *field* theory has instead traditionally been used and confirmed as a method for calculating *S*-matrix elements, which describe probabilities for scattering events. This amounts to applying a collapse of the wave function after each elementary scattering process, and it would be insufficient for consistently describing objects which make up the Universe, such as condensed matter, complex systems (including measurement devices and observers), macroscopic fields, and global spacetime structure.

The general quantum framework is usually applied in the form of a ‘quantization’ of a classical theory (see Sect. 4.1.1) – in particular of the mechanics of particles, which are kinematically described as space points. By quantization I mean here¹ the application of the superposition principle to the elements of a classical configuration space (thus defining a wave function on it), and the construction of the corresponding quantum Hamiltonian by replacing variables and their canonical momenta by operators acting on wave functions. The second part is ambiguous because of the factor ordering problem.

We can now re-interpret this quantization procedure as the conceptual reversal of a physical decoherence process that led to the classical appearance of the system under consideration. This explains why this quantization cannot be expected to define a unique result, but requires further empirical input. The quantization of many-particle mechanics leads non-relativistically ‘back’ to a consistent and successful quantum theory: quantum mechanics. Some other ‘particle’ properties (such as spin or isotopic spin) have *no* similar classical correspondence. The quantization of classical *fields* in this canonical way leads to wave functionals on the configuration space for field amplitudes. It does not in general directly define a consistent quantum theory, although it can often be rendered consistent by a mere renormalization of its fundamental parameters. This is evidence that a fundamental quantum theory may be quite independent of any classical theory that could be quantized in this way. For example, relativistic quantum mechanics led to the discovery that field amplitudes of *not classically observed* fermion fields rather than particle positions define the correct arena for the wave function(al) – an approach that is somewhat misleadingly called a ‘second quantization’, since the fermion fields were first discovered as effective ‘single-particle wave functions’ (see Zeh 2003). The underlying fields (on space) define a local *basis* (the ‘stage’ for quantum

¹ This interpretation is quite different from the original and literal meaning of the term ‘quantization’ as a *discretization* of certain quantities. For example, ‘light quanta’ can be understood as a *consequence* of the eigenvalue problem in terms of wave functions for the amplitudes of free field modes, dynamically described as harmonic oscillators. These fundamental aspects of quantum theory are often hidden behind a collection of recipes to perform calculations (such as perturbation theory in terms of Feynman graphs). In particular, a ‘quantization of time’ (Sect. 6.2) does *not* require a quantum of time – just as the quantization of particle motion does not require a quantum of length (or a spatial lattice).

dynamics) that spans the required Hilbert space. This structure permits the formulation of *local dynamics* by means of a Hamiltonian density in spite of generically nonlocal states. It may therefore be useful – though also dangerous and certainly insufficient – to investigate mathematical models for a unified quantum field theory solely by investigating certain *classical* fields on three- or higher-dimensional spaces, rather than consistently taking into account their quantum nature from the beginning (for instance in terms of wave functionals of these fields as representing the true quantum reality).

Extrapolating unitary dynamics to the whole Universe requires an Everett type interpretation (see Sect. 4.6). Hugh Everett (1957) seems to have first seriously considered a wave function of the Universe,² that must then include *internal observers*. Although he may have had in mind the quantization of general relativity with its cosmological aspects, Everett applied his ideas, which were based on a time-dependent Schrödinger equation, to non-relativistic quantum theory. His main interpretational obstacle was the entanglement arising from measurements described by means of von Neumann’s unitary interaction (4.32). This led him to his ‘extravagant’ interpretation (in Bell’s words) in terms of *many* quasi-classical ‘branches’ of the world, which are separately experienced, but are all assumed to exist in one superposition that defines the true and dynamically consistent quantum world. Beyond measurements proper and occasional interactions he does not seem to have regarded entanglement as particularly important (see Tegmark 1998).

The quantitative considerations reviewed in Sect. 4.3 demonstrate that *uncontrollable* ‘measurement-like’ interactions with the environment are essential and unavoidable for almost all systems under all realistic circumstances. Strong entanglement is, therefore, a *generic* aspect of quantum theory. The more macroscopic a system, the stronger its entanglement with its environment. The concept of a (pure) quantum state can be consistently applied only to the Universe as a whole (Zeh 1970, Gell-Mann and Hartle 1990). This seems to be a far more powerful argument for the need of quantum cosmology than an attempt to construct a unified quantum field theory.

The second pillar of physical cosmology is general relativity. It is empirically confirmed only as a classical theory, but this fact can be well understood by decoherence again (see Sects. 4.3.5 and 6.2.2). Exactly classical gravity would lead to inconsistencies with the uncertainty principle. Applying the quantization rules to the Hamiltonian formalism of general relativity (described in Sect. 5.4) leads to a non-renormalizable ‘effective’ quantum gravity that cannot be exact, but may be expected to be appropriate as a low energy limit. This readily allows us to discuss a number of important novel conceptual problems that must come up, in particular the need for a ‘quantization of time’ (Sect. 6.2).

² Thibault Damour (2006) has recently presented evidence that Everett was originally stimulated by remarks Albert Einstein made about quantum theory during his last seminar, given at Princeton in 1954.

The quantum state of the Universe must therefore include gravitational degrees of freedom (entangled with matter) in an essential way. However, many quantum cosmological aspects may be formulated on a quasi-classical background spacetime, using a given foliation parametrized by a time coordinate t . Global states can then be dynamically described by means of a time-dependent Schrödinger equation with respect to this coordinate time t . This formalism will be *derived* from quantum gravity (with its quantized concept of an *intrinsic* time) in Sect. 6.2.2 as an approximation. *Global* states (such as those of quantum fields) depend on a foliation (or a reference frame) even on flat spacetime, while the density matrix of any *local* system should be invariant under a change of foliation that preserves its local rest frame – a requirement that does not seem to have attracted much attention.

If the Quantum Universe is thus conceptually regarded as a whole, it does not decohere, since there is no further environment. Decoherence is meaningful only for *subsystems* of the Universe (or for subsets of variables), and with respect to observations by other subsystems (internal ‘observer-participants’). If no real collapse of the wave function is assumed to apply, one is then *forced* to accept Everett’s global wave function, which describes a superposition of at least all ‘possible’ outcomes of measurements and measurement-like processes that ever occurred in the Universe. This global quantum state may always be assumed to be pure, since a global density matrix could be consistently understood as representing incomplete information about such a pure state. A measurement that merely selects a subset from those states which diagonalize this density matrix would be equivalent to a classical measurement (as depicted in Fig. 3.5 – in contrast to Fig. 4.3).

The decoherence of subsystems by their environment according to a *global* Schrödinger equation leads dynamically to robust Everett branches. They represent dynamically autonomous *components* of the global wave function, which may factorize in the form $\phi_{\text{obs1}}\phi_{\text{obs2}}\dots\psi_{\text{rest}}$ with respect to ‘observer states’ that may describe objectivizable memory (see Sect. 4.3.2 and Tegmark 2000). This unitary evolution requires a fact-like arrow of time, corresponding to a cosmic initial condition of type (4.59). Branching into components which contain definite observer states has to be *taken into account* in addition to the unitary evolution as an *effective* dynamics in order to describe the history of the (quasi-classical) ‘observed world’ in quantum mechanical terms (see Sect. 4.6 and Fig. 4.3). However, this need *not* represent a modification of the fundamental dynamical laws, since this indeterminism affects the observer rather than the quantum world. The decrease of physical entropy characterizing the ‘apparent collapse’ experienced by the subjective observer may be negligible on a thermodynamical scale, and in comparison to the entropy increase by decoherence in the usual situation of a measurement. Yet it may have dramatic consequences for global phase transitions that describe a dynamical symmetry-breaking of the vacuum. This will now be discussed.

6.1 Phase Transitions of the Vacuum

Heisenberg (1957) and Nambu and Jona-Lasinio (1961) invented the concept of a vacuum that breaks symmetries of a fundamental Hamiltonian ‘spontaneously’ (in a fact-like way) – just as *most* actual states of physical systems do. This proposal was based on an analogy between the vacuum (the ground state of quantum field theory) and the phenomenological ground states of macroscopic systems, such as ferromagnets or solid bodies in general. Their asymmetric ground states lead to specific modes of excitation, which in quantum theory define quasi-particles (phonons, for example). The corresponding occupation number eigenstates span specific partial Hilbert spaces (‘Fock spaces’). A symmetry-violating vacuum may similarly lead to *Goldstone bosons* or other collective modes, based on space-dependent oscillations of the order parameter about its macroscopic (collective) ‘orientation’ – see below.

A symmetry-breaking (quasi-classical) ‘ground state’ is in general not even an eigenstate of the fundamental (symmetric) Hamiltonian; it may only form an eigenstate of an effective (asymmetric) Fock space Hamiltonian. While non-diagonal elements of the exact Hamiltonian which connect states of different collective orientation of these many-body systems (lying in different Fock spaces), are usually extremely small, they would be essential to determine its exact eigenstates, since the diagonal elements for all states related by a symmetry transformation must be degenerate.

The symmetry-breaking vacuum was originally understood as part of the kinematics of a field theory, while the dynamics was then assumed to be completely defined by means of the Fock space Hamiltonian. Later, the analogy was generalized to allow for a *dynamical* phase transition of the vacuum during the early stages of the Universe. This may be induced by the variation of some global parameter (such as a rapid decrease of energy density, reflecting the expansion of the Universe). The arising ‘unitarily inequivalent’ different Fock spaces can then be interpreted as robust Everett branches or collapse components. Even the empirical P or CP -violating terms of the (effective) weak-interaction Hamiltonian may have *emerged* dynamically in this way by means of an apparent or genuine collapse of the wave function that led to a specific vacuum.

A popular model for describing symmetry-breaking in non-perturbative quantum field theory is the ‘Mexican hat’ or ‘wine bottle potential’ of the type $V(\Phi) = a|\Phi|^4 - b|\Phi|^2$ (with $a, b > 0$) for a fundamental complex matter field Φ (such as a *Higgs field*). It may possess a degenerate minimum on a circle in the complex plane, at $|\Phi| = \Phi_0 > 0$, say. The classical field configurations of lowest energy may then be written as $\Phi \equiv \Phi_0 e^{i\alpha}$, with an arbitrary phase α . They break the dynamical symmetry under rotations in the complex Φ -plane. These classical ground states correspond to different quantum mechanical vacuum states $|\alpha\rangle$ (for example described by narrow Gauß packets of α -eigenstates). One of them, $|\alpha_0\rangle$, say, is assumed to characterize our observed world (while the specific value of α_0 is in this case observationally meaningless).

A *dynamical* phase transition of the vacuum can now be described by assuming that the Universe was initially in the symmetric vacuum $|\Phi \equiv 0\rangle$. This may later become a ‘false’ vacuum (a *relative* minimum) through a change of the parameters a and b . The state of the observed universe is then assumed to undergo a transition into a specific Fock space vacuum $|\alpha_0\rangle$. If potential energy is thereby released in a ‘slow roll’ (similar to latent heat in a phase transition), it must be transformed into excitations (particle creation). Evidently, this symmetry-breaking process requires effective deviations from the Schrödinger equation – similar to a measurement process.

If the initial state is here assumed to be pure, a *unitary* evolution (similar to von Neumann’s measurement) leads to a symmetric superposition of all asymmetric states. For example, the symmetric superposition of all Fock space vacua,

$$|0_{\text{sym}}\rangle = C \int |\alpha\rangle d\alpha \neq |\Phi \equiv 0\rangle, \quad (6.1)$$

may possess an even lower energy expectation value than $|\alpha\rangle$, and may thus represent an approximation to the ground state of the full theory. A globally symmetric superposition of type (6.1) would persist even when its components on the RHS contain or develop uncontrollable excitations *in their Fock spaces*, while these components then form dynamically independent Everett branches. The superposition itself describes *intrinsic complexity*, but not a global asymmetry. If $\pi_\alpha := i\partial/\partial\alpha$ generates a gauge transformation, (6.1) describes a state obeying a gauge constraint, $\pi_\alpha|\psi\rangle = 0$ (see Sect. 6.2).

Each homogeneous *classical* state α_0 would permit excitations in the form of small space-dependent oscillations, $\alpha_0 + \Delta\alpha(\mathbf{r}, t)$. Quantum mechanically they describe massless *Goldstone bosons* (excitations with vanishing energy in the limit of infinite wavelength because of the degeneracy). Their degrees of freedom are thus *created* by the intrinsic symmetry breaking, and their observation demonstrates that the collective variables (including corresponding ‘gauge’ degrees of freedom) do not describe mere redundancies. These new variables may be thermodynamically extremely relevant. So it is remarkable that the most important cosmic entropy capacities are represented by zero-mass bosons: electromagnetic and gravitational fields (Zeh 1986a, Joos 1987). These capacities are not only relevant for physical entropy (such as in the form of heat), but also for the formation of entanglement between different spatial regions. This seems to be important for the ‘arrow of quantum causality’ (Sect. 4.6).

In contrast to the false vacuum, the symmetric superposition (6.1) would already describe a nonlocal state. If one neglects Casimir–Unruh type correlations (see Sect. 5.2), each vacuum $|\alpha\rangle$ may be written as a direct product of vacua on volume elements ΔV_k ,

$$|\alpha\rangle \approx \prod_k |\alpha\rangle_{\Delta V_k}. \quad (6.2)$$

This non-relativistic approximation describes a pure vacuum state on each volume element (local subsystem) ΔV_k , while the superposition (6.1) would lead to ‘mixed states’ for them:

$$\rho_{\Delta V_k} \propto \int |\alpha_{\Delta V_k}\rangle \langle \alpha_{\Delta V_k}| d\alpha, \quad (6.3)$$

formally representing Zwanzig projections \hat{P}_{sub} . However, this density matrix would be meaningful only for an *external* observer of the global state (who could not live in one of the Fock spaces). It describes a canonical distribution of Goldstone bosons with infinite temperature (since then $e^{-E/kT} \rightarrow 1$). Therefore, only a (genuine or apparent) collapse into *one* component α_0 gives rise to the pure (cold and not entangled) vacuum (6.2) experienced by an internal local observer who lives in this Fock space.

Order parameters such as α may differ in different spatial regions (similar to Weiss regions of a ferromagnet). If these regions are macroscopic, and thus decohere to become ‘real’ (see Sect. 4.3.1), they break translational symmetry (Calzetta and Hu 1995, Kiefer, Polarski and Starobinsky 1998, Kiefer et al. 2006). This scenario has now become ‘standard’ in quantum cosmology – although its interpretation varies. A homogeneous superposition of entangled *microscopic* inhomogeneities would represent ‘virtual’ symmetry breaking (in classical language circumscribed as ‘vacuum fluctuations’).

6.2 Quantum Gravity and the Quantization of Time

Um sie kein Ort, noch weniger eine Zeit;
Von ihnen sprechen ist Verlegenheit.
(Mephisto advising Faust to time travel)

The compatibility of general relativity and quantum theory has often been questioned. This seems to be a prejudice, that derives from various roots:

Einstein’s attitude regarding quantum theory is well known. He is even claimed to have remarked that a quantization of general relativity would be ‘childish’ – although he also emphasized the importance of reconciling his theory with quantum theory. Another position holds that gravitons may be unobservable in practice, and the quantization of gravity hence *not required* (von Borzeszkowski and Treder 1988). However, a classical gravitational field or spacetime metric is *inconsistent* with quantum mechanics, since it would always allow one in principle to determine the exact energy of a quantum object – in conflict with the uncertainty relations. This has been known since the early Bohr–Einstein debate (see Jammer 1974, for example), while other consistency problems regarding an exactly classical spacetime metric were raised by Page and Geilker (1982). Concepts of quantum gravity will turn out to be essential for cosmology and the definition of a master arrow of time. The classical *appearance* of spacetime cannot be regarded as an argument against