The strange (hi)story of particles and waves*

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Abstract: This is a non-technical presentation (in historical context) of the quantum theory that is strictly based on global unitarity. While the first part is written for a general readership, Sect. 5 may appear a bit provocative. I argue that the single-particle wave functions of quantum mechanics have to be correctly interpreted as *field modes* that are "occupied once" (that is, first excited states of the corresponding quantum oscillators in the case of boson fields). Multiple excitations lead non-relativistically to apparent many-particle wave functions, while the quantum states proper are always defined by wave function(al)s on the configuration space of fundamental fields, or on another, as yet elusive, fundamental local basis.

Contents:

Overview: Sects. 1 and 2 are a brief review of the early history - neglecting details. Sects. 3 and 4 concentrate on some important properties of non-relativistic quantum mechanics that are insufficiently pointed out in many textbooks (including quite recent ones). Sect. 5 describes how this formalism would have to be generalized into its relativistic form (QFT), although this program generally fails in practice for *interacting* fields because of the complicated entanglement that would arise between too many degrees of freedom. This may explain why QFT is mostly *used* in a semi-phenomenological manner that is often misunderstood as a fundamentally new theory. Sect. 6 describes the application of the Schrödinger picture to quantum gravity and quantum cosmology, while Sect. 7 concludes the paper.

* Free and extended translation of my unpublished German text "Die sonderbare Geschichte von Teilchen und Wellen" – available at my website since October 2011. By the term "(hi)story" I tried to catch the double-meaning of the German word "Geschichte". V15 has now been published in Z. Naturf. A, **71**, 195 (2016).
– Depending on your interests, you may prefer to skip the first one or two (historical) sections!

1. Early History

The conceptual distinction between a discrete or a continuous structure of matter (and perhaps other "substances") goes back at least to the pre-Socratic philosophers. However, their concepts and early ideas were qualitative and speculative. They remained restricted to some general properties, such as symmetries, while the quantitative understanding of continuous matter and motion had to await the conceptual development of calculus on the one hand, and the availability of appropriate clocks on the other. Quantitative laws of nature and the concept of mass points, for example, were invented as part of classical mechanics.

This theory was first applied to extended "clumps of matter", such as the heavenly bodies or falling rocks and apples. It was in fact a great surprise for Newton and his contemporaries (about 1680) that such very different objects – or, more precisely, their centers of mass – obeyed the same laws of motion.[1] The objects themselves seemed to consist of continuous matter, although the formal concept of mass points was quite early also applied to the *structure* of matter, that is, in the sense of an atomism. Already in 1738, Daniel Bernoulli explained the pressure of a gas by the mean kinetic energy of presumed particles, but without recognizing its relation to the phenomenon of heat. If one regarded these particles themselves as small elastic spheres, however, the question for their internal structure would in principle arise anew. The concept of elementary particles thus appears problematic from the outset.

At about the same time, Newton's theory was also generalized by means of the concept of a continuum of infinitesimal mass points which can move according to their local interaction with (mainly their repulsion by) their direct neighbors. This route to continuum mechanics required novel mathematical concepts, but no fundamentally new *physical* ones beyond Newton. The assumption of an unlimited divisibility of matter thus led to a consistent theory. In particular, it allowed for wave-like propagating density oscillations, required to describe the phenomenon of sound. So it seemed that the fundamental question for the conceptual structure of matter had been answered.

As a byproduct of this "substantial" (or "Laplacean") picture of continuum mechanics, based on the assumption of distinguishable and individually moving infinitesimal elements of matter, also the elegant "local" (or "Eulerian") picture could be formulated. In the latter, one neglects any reference to trajectories of individual elements in order to consider only its spatial density distribution together with a corresponding current density as the kinematical objects of interest. In modern language they may be called a scalar and a vector *field*. In spite of this new form, however, continuum mechanics remained based on the concept of a locally conserved material substance consisting of individual elements.

This model for a continuum of mass points would be incomplete if the latter could move freely, interrupted only by occasional collisions, as suspected for a gas by Daniel Bernoulli. Since his gas pressure (which allows for sound waves, too) is given by the density of molecular kinetic energy, that is, by the product of the number density of gas particles and their mean kinetic energy, this could still be understood as representing a "chaotic continuum" by means of an appropriately defined simultaneous limit of infinite particle number density and vanishing particle size. This remained a possibility even when chemists began to successfully apply Dalton's and Avogadro's hypotheses about molecular structures from the beginning of the nineteenth century in order to understand the chemical properties of the various substances. Similar to Auguste Bravais's concept of crystal lattices (about 1849), these structures were often regarded as no more than a heuristic tool to describe the internal structure of a multi-component continuum. This view was upheld by many even after Maxwell's and Boltzmann's explanation of thermodynamic phenomena in terms of molecular kinetics, and in spite of repeated but until then unsuccessful attempts to determine a finite value for Avogadro's or Loschmidt's numbers. The "energeticists", such as Wilhelm Ostwald, Ernst Mach and initially also Max Planck remained convinced until about 1900 that atoms are an illusion, while concepts like internal energy, heat and entropy would describe fundamental continua (fields). Indeed, even after the determination of Loschmidt's number could they have used an argument that formed a severe problem for atomists: Gibbs' paradox of the missing entropy of self-mixing of a gas. Today it is usually countered by referring to the *indistinguishability* of molecules of the same kind, although the argument requires more, namely the *identity* of states resulting from their permutations. Such an identity would be in conflict with the concept of particles with their individual trajectories, while a *field* with two bumps at points x and y would by definition be the *same* as one with bumps at y and x Although we are using quite novel theories today, such conceptual subtleties do remain essential – see Sect. 5. (Their role in statistical thermodynamics depends also on dynamical arguments.)

Another object affected by the early dispute about particles and waves is light. According to its potential of being absorbed and emitted, light was traditionally regarded as a "medium" rather than a substance. Nonetheless, and in spite of Huygens' early ideas of light as a wave phenomenon in analogy to sound, Newton tried to explain it by means of "particles of light", which were supposed to move along trajectories according to the local refractive index of matter. This proposal was later refuted by various interference experiments, in particular those of Thomas Young in 1802. It remained open, though, what substance (called the ether) did oscillate in space and time – even after light had been demonstrated by Heinrich Hertz in 1886 to represent an electromagnetic phenomenon in accordance with Maxwell's equations. The possibility of these fields to propagate and carry energy gave them a certain substantial character that seemed to support the world of continua as envisioned by the energeticists. Regarding atoms, Ernst Mach used to ask "Have you ever seen one?" whenever somebody mentioned them to him. Later in this article I will argue that his doubts may be justified even today – although we *seem* to observe individual atoms and particle tracks. Similar to the phenomenon of "events" or "quantum jumps", they may be an illusion caused by the dynamics of Schrödinger's wave function, which does *not* live in space (Sect. 3).

At the end of the nineteenth century, the continuum hypothesis suffered a number of decisive blows. In 1897, J. J. Thomson discovered the elementary electric charge; in 1900, Max Planck postulated his radiation quanta for the electromagnetic field with great success; and in 1905, Albert Einstein estimated the value of Loschmidt's number N_L by means of his theory of Brownian motion. Thereafter, even the last energeticists resigned, but they left some confusion about the concept of a physical "state". While they had regarded temperature, pressure or internal energy density etc. as locally characterizing the ontic state of matter, in atomistic description these "thermodynamic states" require some averaging over the fundamental particle states, either in time, or in space ("coarse graining"), or with respect to some incomplete knowledge. In quantum theory, this confusion survives in the operationalist definition of states and in the concept of "mixed states" (see Sect. 4).

Einstein even revived the concept of particles of light (later called photons) – although he regarded it merely as a "heuristic point of view" that he was never ready to fully accept himself. For a long time, Planck's radiation quanta were indeed attributed to a discrete emission process rather than to the radiation itself. So in 1913, Niels Bohr replaced the concept of classical motion for atomic electrons by stochastic "jumps" between his discrete atomic orbits – in accordance with Planck's and Einstein's ideas about a probabilistic radiation process. These early ideas led later to the insufficient interpretation of quantum mechanics as no more than stochastic dynamics for otherwise classical particles.

However, the development soon began to proceed in the opposite direction again.[2] In 1923, Louis de Broglie inverted Einstein's speculative step from light waves to photons by postulating a wave length $\lambda = c/\nu = h/p$ for the electron, where p is its momentum, in analogy with Planck's relation $E = pc = h\nu$. For him, this could only mean that all microscopic objects must consist of both, a particle *and* a wave, whereby the wave has to serve as a "guiding field" or "pilot wave" for the particle. This field would have to be more powerful than a conventional *force* field, since it has to determine the velocity rather than merely the acceleration; the initial velocity can according to this proposal not be freely chosen any more once the wave function is given. When David Bohm later brought this theory into a consistent form, it turned out that the pilot wave cannot be defined in space ("locally"), since it has to be identified with the global entangled wave function to be described in Sect. 4.

2. Wave Mechanics

Inspired by de Broglie's ideas, Schrödinger based his novel wave mechanics of 1926 on the assumption that electrons are *solely* and uniquely described by wave functions (spatial fields, as he first thought). His wave equation allowed him to explain the hydrogen spectrum by replacing Bohr's specific electron orbits by standing waves. In this way he could explain the puzzling discrete quantum numbers by the numbers of nodes the wave function needs to obey its boundary conditions. For a special case (the harmonic oscillator) he was furthermore able to construct "wave packets" that may imitate *moving* particles – see Fig. 1 for the case of *free* motion, however. Shortly thereafter, interference phenomena in agreement with de Broglie's wave length were observed by Davisson and Germer for electrons scattered from crystal lattices. A wave function can furthermore penetrate a potential barrier and thus quantitatively explain "quantum tunneling" for the phenomenon of α-decay. Does this not very strongly indicate that electrons and other "particles" are in reality just wave packets of fields that obey Schrödinger's wave equation?

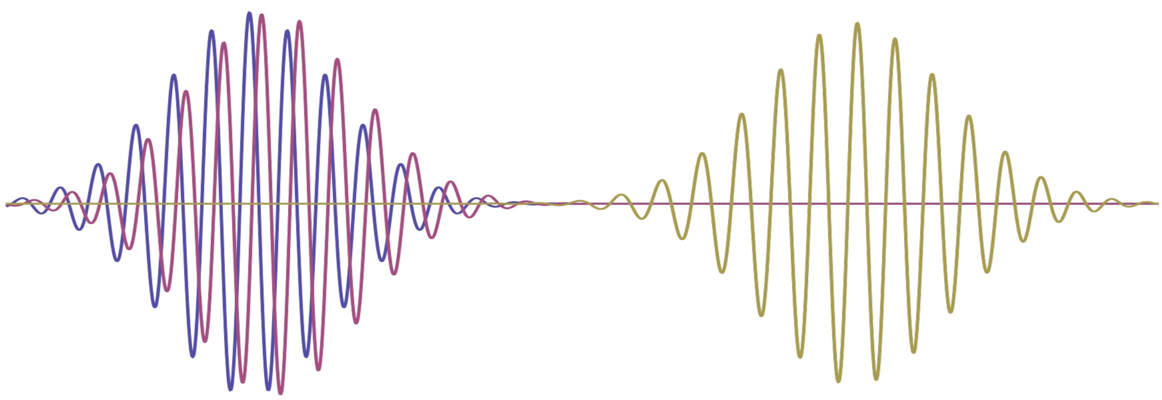

Fig. 1: Real part of a one-dimensional complex wave packet (the product of a Gaussian with a plane wave $e^{2\pi i x/\lambda}$) moving freely according to the time-dependent Schrödinger equation, depicted at three different times (blue: t=0,

red: t=0.04, yellow: t=1 in relative units). If the wave packet describes reality, its width defines a "real uncertainty" for the object position; it does neither represent incomplete information, nor is it related to the measureable "particle" size (which has to be described by *internal* degrees of freedom – see Sect. 4). When comparing blue and red, one recognizes that the packet moves faster than its wave crests, while the yellow curve demonstrates a slight spreading of the packet (in contrast to the mentioned harmonic oscillator). The center of the packet moves according to the group velocity $v = p/m := h/m\lambda$, where the mass m is just a parameter of the wave equation. For this reason, momentum is in wave mechanics *defined* by the wave number h/λ (not by motion!), although it is mostly *observed* by means of moving wave packets (moving "objects"). It can then be measured even for plane waves, which would not define a group velocity, by means of the conservation of wave numbers $k = 2\pi/\lambda$ during interactions with objects that do exist as wave packets, thus giving rise to the concept of "momentum transfer". Already for atomic masses and thermal velocities, the de Broglie wave length is clearly smaller than the radius of a hydrogen atom, so one may construct quite narrow wave packets for their center of mass (cms) wave functions. While the dispersion of the wave packet decreases with increasing mass m, it becomes always non-negligible after a sufficient time interval. In order to compensate for it, one would need an additional dynamical mechanism that permanently reduces the "coherence length" characterizing a wave packet in order to retain the appearance of a particle (see for "collapse" or "decoherence" in Sect. 4).

A few months before Schrödinger invented his wave mechanics, Heisenberg had already proposed his matrix mechanics. In contrast to Schrödinger, he did not abandon the concept of particles, but in a romantic attempt to revive Platonic idealism and overcome a mechanistic world view, combined with an ingenious guess, he introduced an abstract formalism that was to replace the concept of deterministic trajectories by formal probabilistic rules. Together with Born and Jordan, Heisenberg then constructed an elegant algebraic framework that could be used to "quantize" all mechanical systems. This mathematical abstraction perfectly matched Heisenberg's idealistic philosophy. In particular, matrix mechanics was shown *in principle* to lead to the same predictions as wave mechanics – although it could be used in practice only in simple cases. A year after his first paper, Heisenberg supplemented his formalism by his uncertainty relations between position and momentum of an electron or other "conjugate" pairs of variables. Such a fundamental uncertainty is clearly in conflict with a consistent concept of particles, while in wave mechanics it would simply be a consequence of the Fourier theorem – without any uncertainty *of the wave function* or the assumption of an unavoidable "distortion" of the state of the electron during a measurement (as originally suggested by Heisenberg). Another indication of a choice of inappropriate concepts may be the requirement of a "new logic" for them. So it is not surprising that Schrödinger's intuitive wave mechanics was preferred by most atomic physicists – for a short time even by Heisenberg's mentor Max Born. For example, Arnold Sommerfeld wrote only a "Wellenmechanischer Ergänzungsband" to his influential book "Atombau und Spektrallinien".

Some important phenomena, though, remained in conflict with Schrödinger's theory. While his general wave equation $i\hbar\partial\psi/\partial t = H\psi$ would allow various time-dependent solutions, such as the moving wave packet of Fig. 1, bound electrons appeared to be restricted to standing waves. The latter are solutions of the stationary Schrödinger equation $H\psi = E\psi$ that gives rise to the observed discrete eigenvalues E_n under the required boundary conditions. Although this equation can be derived from the general one under the assumption of a special time dependence of the form $\psi \propto e^{iEt/\hbar}$, there is no obvious reason for this special form. Instead of obeying the time-dependent equations, these bound states seemed to be dynamically related by Bohr's stochastic "quantum jumps", which would thus explain energy quanta of radiation (including the hydrogen spectrum) by means of the conservation of energy. Other wave functions seem to "jump" or "collapse" into particle-like narrow wave packets during position measurements. In a Wilson chamber, one could even observe tracks of droplets that can be regarded as successions of such position measurements along particle trajectories.

As a consequence, Schrödinger seemed to resign when Max Born, influenced by Wolfgang Pauli, re-interpreted his new probability postulate, which originally was to describe jumps between different wave functions, in terms of probabilities for the spontaneous *creation of particle properties* (such as positions or momenta). This interpretation turned out to be very successful (and earned Born a Nobel prize) even though it was never quite honest, since the wave function does *not only* describe probabilities. It is also required to represent individual observable properties, such as energy or angular momentum, by means of corresponding "eigenstates", whose spatial structure can often be confirmed by appropriate experiments. Similarly, a spinor (a generalized wave function for the electron spin) describes probabilities for other *individual* spinor states rather than for classical properties.

The impossibility to derive the successful wave function from his uncertainty principle (while the reverse *is* possible) was so painful for Heisenberg that he regarded the former as "a new form of human knowledge as an intermediary level of reality", while Bohr introduced his, in his own words, "irrational" principle of complementarity. It required the application of mutually exclusive ("complementary") classical concepts, such as particles and waves, to the same objects. No doubt – this was an ingenious pragmatic strategy to avoid many problems, but, from there on, the search for a consistent description of Nature was not allowed any more in microscopic physics. Pure *Gedanken*-experiments, traditionally used as consistency tests for physical concepts, were now discredited for being "counterfactual". As an answer to the question whether the electron be *really* a wave or a particle (or what else), Bohr insisted that "there is no microscopic reality" – a conclusion that was often regarded as philosophically very deep. Only few dared to object that "this emperor is naked", and the term "complementarity" no more than a new name for a conceptual inconsistency. The large number of attempts of a philosophical or formal explanation of this "nonconcept" is even the more impressive. Furthermore, the question when and where precisely the probability interpretation (or the "Heisenberg cut" between quantum and classical concepts) has to be applied, that is, when a "virtual" property becomes "real", remained open to be pragmatically decided from case to case. Therefore, the Hungarian Eugene Wigner spoke of a "Balkanization of physics" – a traditional (Hapsburgian) expression for the decay of law and order during that time.

3. Wave Functions in Configuration Space

So one should take a more complete look at Schrödinger's wave mechanics. When he formulated it, he used Hamilton's partial differential equations as a guiding principle. These equations, the result of a reformulation of classical mechanics, are solved by a scalar function whose gradient describes a continuum of independent classical trajectories which differ by their initial conditions – sort of a wave function without interference. Hamilton had mainly been interested in the elegant mathematical form of this theory rather than in applications. This turned out to be an advantage for Schrödinger. He assumed that Hamilton's equations were no more than a short wave lengths approximation (corresponding to the limit $h \to 0$) of a *fundamental* wave theory – similar to the approximation of geometric optics in Maxwell's theory. However, this short wave length approximation only means that local parts of an extended wave propagate almost independently of one another along spatial paths – not that they represent particles. Similarly, Feynman's path integral defines a propagating *wave* as a superposition of the various causal chains contained in such a continuum,[3] while it neither requires nor justifies the existence of individual paths that might then simply be selected by an increase of information. Different partial waves or Feynman paths can in fact interfere with one another (that is, they may have coherent physical effects). This means that they exist together as *one reality* (one wave function) rather than merely defining a statistical ensemble of *possibilities*. They could be turned into an ensemble only by some stochastic dynamics that would have partially to replace the deterministic wave equation.

While light waves propagate in three-dimensional space, Hamilton's waves must according to their construction exist in the configuration space of all possible classical states q

of the system under consideration. Therefore, Schrödinger, too, obtained wave functions on (what appears to us classically as) configuration spaces of various dimensions rather than in space. This is an enormous difference, that turned out to be very important for atoms and molecules. Intuitive *spatial* wave functions are here quite insufficient, in general. The new wave functions can also be understood as a consequence of Dirac's fundamental superposition principle, since the superposition of all classical configurations q defines precisely a wave function $\psi(q)$ on configuration space. It can then easily be further generalized to include properties that never occur as classical variables (such as spin, neutrino flavor, or even the difference between a K-meson and its antiparticle), whose superpositions may again define new *individual* physical states (even new kinds of "particles"). Dirac himself understood his superpositions in Born's pragmatic but still enigmatic sense as "probability amplitudes" for properties that are formally represented by Heisenberg's classically motivated "observables". There is no absolutely preferred basis in Hilbert space, and probabilities are thus meaningful only with respect to corresponding "measurements". If these observables are written in terms of dyadic products of their eigenstates (their spectral representation), they may formally describe Born's probabilities as those for *jumps of wave functions* (stochastic projections in Hilbert space as part of the dynamics). Any proposal for some fundamental theory underlying quantum mechanics would first of all have to explain the very general and well established superposition principle, which, in particular, describes all phenomena of quantum nonlocality without any "spooky" action at a distance (see Sect. 4).

Schrödinger was still convinced of a reality in space and time, and so he initially hoped, in spite of the Hamiltonian analogy, to describe the electron as a spatial *field*. Therefore, he first restricted himself with great success to single-particle problems (quantized mass points, whose configuration space is isomorphic to space). Consequently, he spoke of the "ψ-field". This approach misled not only himself, but a whole generation of physicists. A spatial wave function can also be readily used to describe scattering problems – either applied to the center-of-mass wave function of an object scattered from a potential, or to the relative coordinates of a two-body problem. In scattering events, Born's probability interpretation is particularly suggestive because of the usual subsequent position measurement in a detector. A wave function in space is indeed usually meant when one speaks of the *wave-particle dualism*. In spite of its limited and therefore misleading value, three-dimensional wave mechanics still dominates large parts of most textbooks because of its success in correctly and simply describing many important single-particle aspects, such as the energy spectrum of the hydrogen atom and scattering probabilities. It is often supported by presenting the two-slit experiment as *the* key to understanding quantum mechanics, although this is only one specific aspect.

The generalization (or rather the return) to wave functions in configuration space happened almost unnoticed at those times of great confusion – for some physicists even until today. While most of them are now well aware of "quantum nonlocality", they remain used to arguing in terms of spatial waves for many purposes. In contrast to classical fields, however, single-particle wave functions do not describe additive (extensive) charge or energy distributions, since each piece cut from a plane wave representing a quantum "particle", for example, would describe its full charge and kinetic energy (the latter defined by the wave number).

Initially, Schrödinger took great pains to disregard or to re-interpret his general wave equation in configuration space, even though it is precisely its application to oscillating field amplitudes rather than mass points that explains Planck's radiation quanta $h\nu$. (Another early example is the rigid rotator, whose wave function depends on the three Euler angles.) The spectrum $E = nh\nu$ that one obtains for quantum oscillators q_i (here the amplitudes of fixed field modes rather than mechanical mass points), which classically oscillate in time with different frequencies ν_i, is proportional to the natural numbers n. Only this specific spectral property admits the concept of additive *energy quanta* $h\nu$ – later identified with photons – regardless of any emission process that had often been made responsible for their existence. In this way it also explains the concepts of "occupation" or "particle" number. In Schrödinger's wave mechanics, these quantum numbers n can again be explained by the numbers of nodes of the corresponding wave functions. The latter have to be distinguished from the given field modes with their fixed *spatial* nodes, such as $sin(\boldsymbol{k}_i\boldsymbol{x})$ multiplied with a polarization vector. These field modes (rather than their wave functions) can then be regarded as "photon wave functions" – see below and Sect. 5.

But where can one find these oscillator wave functions if not in space? In contrast to the figure, they are here defined as functions on the abstract configuration space of field amplitudes q_i. Different eigenmodes of a *classical* field $q(\boldsymbol{x},t)$, such as plane waves with their classical wave numbers $k_i = 2\pi\nu_i/c$, can fortunately be quantized separately; their Hamiltonians commute. This means that energy eigenstates Ψ for the total quantum field factorize in the form $\Psi=\Pi_i\psi_i(q_i)$, while their eigenvalues simply add, $E = \Sigma_i E_i$. Although the oscillator spectrum $E_i = n_i h\nu_i$ can also be derived from Heisenberg's algebra of observables (matrix mechanics) without explicitly using wave functions, the latter's nodes for a fixed field mode q_i have recently been made visible and thus confirmed for various "photon number" eigenstates (similar to different energy eigenfunctions of the electron in the hydrogen atom) in an elegant experiment.[4] The wave functions $\psi_i(q_i)$ on configuration space have thus been shown to "exist", although they cannot be attributed to the traditional wave-particle dualism, which would refer to *spatial* waves characterizing "quantum particles". The importance of this fundamental experiment for the wave-particle debate has in my opinion not yet been appropriately appreciated by the physics community or in textbooks (see Sect. 5 for further details).

The difference between Schrödinger's theory and a classical field theory becomes particularly obvious from the fact that the amplitudes of a classical field now appear as *arguments q* in Schrödinger's wave function. Positions occur here only as an "index" that distinguishes field amplitudes at different space points, where they form a spatial continuum of *coupled* oscillators. Since classical fields are usually written as functions on space and time, $q(\boldsymbol{x},t)$, the confusion of their spatial arguments with particle positions in the single-particle wave function $\psi(\boldsymbol{x},t)$ led to the questionable concept of a "time operator" to establish some symmetry of space and time. However, $\boldsymbol{x}$ and t in the field q are both classical coordinates, while the particle position $\boldsymbol{x}$ in ψ defines dynamical degrees of freedom (still called "variables" although they now appear only as arguments of the time-dependent wave function).

While a general time-dependent "one-photon wave function" can be understood as a quantum superposition of different modes of the electromagnetic field (such as different plane waves) that are in their first excited quantum state ("occupied once" – with all others in their ground state), a quasi-classical *field* state has in QFT to be described as a coherent superposition of many *different* excitations $\psi_i^{(n)}(q_i,t)$ (different "photon numbers" n) for each spatial eigenmode i. In contrast to the *free* wave packet shown in Fig. 1, these "coherent oscillator states" (time-dependent Gaussians, now functions of the field amplitude) preserve their shape and width *exactly*, while their centers follow classical trajectories $q_i(t)$. Therefore, they imitate oscillating classical fields much better than wave packets in space may imitate particles.

One and the same quantum concept of field functionals Ψ may thus represent "complementary" classical concepts such as "particle" numbers and field amplitudes (albeit again mutually restricted by a Fourier theorem). For this reason, the *Boltzmann distribution* $e^{-E/kT}$ of their energy eigenstates may describe the Planck spectrum with its particle and wave limits for short and long wavelengths, respectively. Field functionals can also describe all specific phenomena of quantum optics, such as "photon bunching".

4. Entanglement and Quantum Measurements

Before trying to study *interacting* quantum fields (Sect. 5), early quantum physicists successfully investigated the quantum mechanics of non-relativistic many-particle systems, such as multi-electron atoms, molecules and solid bodies. These systems could often *approximately* be described by means of different (orthogonal) single-particle wave functions for each electron, while the atomic nuclei seemed to possess fixed or slowly moving positions, similar to classical objects. For example, this picture explained the periodic system of the chemical elements. On closer inspection it turned out, however – at first for the ground and excited states of atoms and small molecules – that *all N* particles forming such objects, including the nuclei, have to be correctly described by one common wave function in their *3N*-dimensional configuration space. This cannot normally be a product or determinant of single-particle wave functions – a consequence that was later called "entanglement". It must similarly apply to different wave modes q_i of interacting fields in QFT. Given any two systems, the set of all their separating (non-entangled) states must have measure zero. When David Bohm began to study consequences of this fundamental property for his theory of 1952, he referred to it as "quantum wholeness", since it means that quantum theory can *only* be consistently understood as quantum cosmology (see Sect. 6). Historically, the essential role of this generic entanglement was often belittled as a mere statistical correlation between subsystems, while this misinterpretation was then further used to argue against an ontic interpretation of the wave function – although entanglement must evidently be a generic part of reality. The presently very popular toy model of entangled qubits, interrupted by classically described actions of Alice and Bob, is no more than an inconsistent caricature of quantum mechanics.

Every physics student is using the entanglement between an electron and a proton in the hydrogen atom when writing the wave function as a product of functions for center-of-mass and relative coordinates. This would not make sense for interacting classical fields, which always *exist* separately. While the wave function for the relative coordinates then defines the size and shape of the hydrogen atom, the center of mass may be represented by a free spatial wave packet as in Fig. 1. The simplest nontrivial case of entanglement, the Helium atom, was first successfully studied in great numerical detail by Hylleraas, using variational methods, in a series of papers starting in 1929. Already Arnold Sommerfeld noticed in his *Wellenmechanischer Ergänzungsband* that "Heisenberg's method", which used only the anti-symmetrization of product wave functions by means of "exchange terms", is insufficient to describe multi-particle systems. (Anti-) symmetrisation is indeed often confused with entanglement, since it is formally equivalent to an entanglement between physical variables and meaningless particle numbers. It merely eliminates any concept of distinguishability, and it is therefore not required any more in the occupation number representation of QFT (see Sect. 5).[†] Genuine entanglement in microscopic systems means, for example, that one has to take into account "configuration mixing" as a correction to the independent-particle (Hartree-Fock or mean field) approximation. For long-range interactions, entanglement may be small in the ground state, since according to the independent-particle picture it would require "virtual excitations" (which are often misinterpreted as "fluctuations" rather than static entanglement).

An important consequence of entanglement is that subsystem Hamiltonians are in general not (or not uniquely) defined – thus ruling out *local* unitarity or a uniquely defined Heisenberg or interaction picture for open systems. *Closed* non-relativistic N-particle systems, on the other hand, can be described by *one* wave function in their complete configuration space – but in practice hardly in the Heisenberg picture. Their center-of-mass wave functions may then factorize from the rest, thus leading to free *spatial* wave functions for them (identical to those for mass points or "quantum particles"), while the internal energy quantum numbers are given by the number of nodes (or zeros: now defining hypersurfaces) in the remaining $3(N-1)$-dimensional configuration space. For non-inertial motion, this separability is only approximately valid – depending on the required internal excitation energy.[5] Isolated systems formed the major objects for studying quantum mechanics, although they represent an exception. Open system quantum mechanics was studied much later – mostly in a semi-phenomenological manner, and in combination with statistical physics. Time-dependent Hamiltonians form a classical concept, as they require time-dependent *states as a source* (thus neglecting their entanglement). When unitary dynamics was consistently applied to global systems in order to derive subsystem dynamics, it led to the phenomenon of decoherence.

However, how can the space of all possible classical configurations, which would even possess varying dimensions, replace three-dimensional space as the true fundamental arena for the dynamics of physical states? If our Universe consisted of N particles (and nothing else), its configuration space would possess $3N$ dimensions – with N being at least of the order 10^{80}. For early quantum physicists – including Schrödinger, of course – such a wave function was inconceivable, although the concept of a space of *possible configurations* fits excel-

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† *Separate* (anti-)symmetrization of spin and orbit parts, however, may define *physical* entanglement; in atomic physics, for example, one has to antisymmetrize in jj-coupling if one wants to avoid that. The statement "particle at position x_1" (in contrast to "particle number 1") "has spin-up" – as in a Bell type experiment – *is* physically meaningful.

lently with Born's probabilities for classical properties. Entanglement can then conveniently be understood as describing statistical correlations between measured variables. But only between *measured* variables! Since macroscopic variables are "permanently measured" by their environment (see below for decoherence), their entanglement almost always appear as no more than a statistical correlation. Only this explains why we are used to interpret the space on which the wave function is defined as a "configuration" space. In bound microscopic systems, however, entanglement is responsible for the precise energy spectrum and other *individual* properties – regardless of any statistical interpretation. This conceptual difference is often simply "overlooked" in order to keep up the illusion of an epistemic interpretation of the wave function (where probabilities would reflect incomplete information about some unknown real variables). Even in individual scattering events one often needs entangled scattering amplitudes with well defined phase relations between all fragments, when mere scattering *probabilities* would be insufficient. Only after Einstein, Podolsky and Rosen (EPR) had shown in 1935 that the entanglement between two particles at a distance may have directly observable consequences, did Schrödinger regard this property as the greatest challenge to his theory – although he kept calling it a "statistical correlation". EPR had indeed erroneously concluded from their analysis that quantum mechanics cannot represent a complete description of Nature, so that as yet unknown ("hidden") variables should be expected to exist.

While many physicists speculated that such hypothetical hidden variables might never be observed in an experiment (even though they might exist), it came as a surprise to them when John Bell showed in 1964 that *any* kind of hidden local reality (no matter whether it consists of particles, fields or other local things with local interactions only – observable or not) would be in conflict with certain observable consequences of entangled wave functions. This conclusion eliminated the major argument for an epistemic interpretation of the wave function. In order to prove his theorem, Bell used arbitrary local variables λ (just a name for something not yet known) for an indirect proof. However, most physicists had by then become so much accustomed to Bohr's denial of a microscopic reality that they immediately accused Bell for having used a "long refuted assumption". The Copenhagen interpretation does indeed clearly go beyond a merely epistemic understanding of the wave function, since, insofar as it refers to ensembles at all, the latter are only meant in a purely formal sense – not in terms of any *elements* (those hidden variables) which would in principle answer the question "Information about what?" In this "operational" approach (supported by Günther Ludwig, for example), the essential question is therefore simply swept under the carpet.

Crucial direct tests of this quantum nonlocality had in practice to be restricted to two- or few-particle systems, which can be isolated from anything else until they are measured. While their entanglement, as a direct consequence of the superposition principle, has thereby always been confirmed, physicists are still debating whether this fact excludes locality (in three-dimensional space) or any kind of microscopic *reality*. For neither those who accept reality to be described by a nonlocal wave function nor those who deny *any* microscopic reality feel particularly bothered by Bell's theorem. These two camps usually prefer the Schrödinger picture (in terms of wave functions) or the Heisenberg picture (in terms of observables), respectively, and this seems to be the origin of many misunderstandings between them. In the absence of any local *states*, the locality of dynamics ("relativistic causality") may appear even difficult to define – but see the discussion in the third paragraph from the end of Sect. 5.

If one does assume the superposition principle to apply universally, one is forced to accept one entangled wave function for the whole universe. Heisenberg and Bohr assumed instead that the wave function is no more than a calculational tool, which "loses its meaning" after the final measurement that concludes an experiment. This "end of the experiment" (related to the "Heisenberg cut") remains vaguely defined and *ad hoc*. Its traditional application (namely, too early in the chain of interactions that leads to an observation) had indeed delayed the discovery of decoherence, which will be discussed below, for several decades. A universal wave function that always evolves according to the Schrödinger equation, however, leads to an entirely novel world view that, in spite of being consistent, appears quite inacceptable to many physicists.

For example, if one measures a microscopic object that is initially in a superposition of two or more different values of the measured variable, this gives rise to an entangled state for the microscopic system and the apparatus – the latter including Schrödinger's infamous cat if correspondingly prepared. (All unitary interactions discussed here and below can be assumed to be of a form like $(\sum_n c_n \psi_n)\Phi_0 \rightarrow \sum_n c_n \psi_n \Phi_n$, that is, transforming local superpositions into entanglement – in the "ideal" case without changing or "disturbing" the measured states ψ_n.) Since such superpositions have never been observed, one traditionally assumes, according to von Neumann, that Schrödinger's dynamics has to be complemented by a stochastic "collapse of the wave function" into one of these components, that is, into a product of narrow wave packets for macroscopic or mesoscopic variables (such as pointer positions Φ_n). Note that, in the Schrödinger picture, Heisenberg's "observables" are readily defined (up to a scale) by the interaction between system and apparatus rather than forming an independent ingredient of the theory. Since this interaction characterizes a measurement device regardless of the time of its application, it appears physically entirely unreasonable to endow it with the actual dynamics *of the object* (as done in the Heisenberg picture). In the Copenhagen interpretation, one would therefore pragmatically jump from a description in terms of wave functions to one in classical terms, and back to a new wave function in order to describe a subsequent experiment. This unsatisfactory situation is known as the *quantum measurement problem*.

If one is ready, instead, to accept a universal Schrödinger equation for describing reality, one must try to understand what an entangled wave function for the microscopic system plus an apparatus might mean. Toward that end one has to include the observer into this description.[6] When he reads off the measurement result, he becomes himself part of the entanglement. According to the unitary dynamics, he would thereafter simultaneously exist in different states of awareness (different states of mind) – similar to the fate of Schrödinger's cat. Hugh Everett first dared to point out in 1957 that this consequence is not in conflict with our subjective observation of *one* individual outcome, since each arising "component state" (or "version") of the observer can register and remember (hence be aware of) only that outcome which is realized in his corresponding "relative state" of the world. The latter would then also contain only consistent versions of all the observer's "friends" – thus defining objectivized outcomes. As there are many such correlated component states (with many minds) in one *global* superposition, though, the question which of them contains the physicist who prepared the experiment has no unique answer; according to the unitary dynamics they all do.

However, why can these components be regarded as separate "worlds" with separate observers? The answer is that they are dynamically "autonomous" after an irreversible measurement in spite of their common origin; each of them describes a quasi-classical world for its macroscopic variables (see the discussion of decoherence below). In contrast to identical twins, who also have one common causal root, different versions of the "same" observer in autonomous branches cannot even communicate any more according to the unitary dynamics, and thus can conclude each other's existence only by an extrapolation by means of the dynamical laws they may happen to know. This is certainly an unconventional, but at least a consistent picture, and a straightforward consequence of the Schrödinger equation. It only requires an unconventional identification of subjective states of individual observers that is consistent with a nonlocal wave function under local interactions. Attempts to avoid this conclusion are all motivated by traditional expectations, and they lead back to an unsolved measurement problem.

Until recently one preferred to believe, instead, that some conceptual or dynamical border line between micro- and macrophysics must exist – even though it could never be located in an experiment. Otherwise it should be possible (so it seemed) to observe individual consequences of entanglement between microscopic systems and their macroscopic measurement instruments – similar to the energy or other properties of Hylleraas's entangled Helium atom or of small molecules. However, this bipartite entanglement is not yet complete. Macroscopic systems must inevitably, extremely fast, and in practice irreversibly interact with their natural "environment", whereby the entanglement that had resulted from the measurement proper would uncontrollably spread to include much of the "rest of the universe". This happens even before an observer possibly enters the scene. In this way, one may understand how a superposition that extends over different macroscopic pointer positions, for example, would, from the point of view of a potential local observer, inevitably be transformed into an effective ensemble of narrow wave packets that mimic classical states (points in configuration space) as potential outcomes. While still forming one superposition, all these partial waves, which must each include different versions of all observer's "friends", have no chance to meet again in high-dimensional configuration space in order to have local coherent consequences. In this sense only, they can now be *regarded as forming an ensemble* of different "worlds".

This unavoidable entanglement with the environment (whose onset defines the true border line between micro- and macrophysics) is called decoherence,[7] as predominantly phase relations defining certain quantum mechanical superpositions become unavailable – that is, they are irreversibly "dislocalized".[‡] As Erich Joos and I once formulated it, the superposition

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[‡] A mere phase *randomization* ("dephasing") could neither occur under unitary evolution, nor would it solve the issue, as each individual member of an ensemble of superpositions with different phases would remain a superposition (though possibly with *unknown* phase). Similarly, local phases that are assumed to fluctuate rapidly in time for some reason are in a definite superposition at any instant. Nonetheless, phase averaging forms the most popular misunderstanding of decoherence, which describes entanglement with an uncontrollable environment in the *individual* case (no averaging). These different concepts are easily confused, in particular, if the environment is described as a "thermal bath". However, if this initial thermal "mixture" had been caused by earlier *quantum* interactions with the environment (which is its most plausible origin), the thus pre-existing entanglement would simply be dynamically extended to the "dephased" variables, where it would then again lead to their genuine decoherence (a dislocalization of their *individual* relative phases). Using the reduced density matrix formalism for this purpose would instead tacitly replace nonlocal entanglement by local ensembles: entanglement is ill-defined for "mixed states". It is remarkable that many important physicists are still missing the essential point of decoherence as a consequence of the fundamental nonlocality of (pure) quantum *states*. Nonlocal phase relations may even be required to define observable individual properties (such as the total spin of two particles at very different positions) in spite of contributing to decoherence. – Historically, the term "decoherence" was first invented in the context of "decoherent histories" in about 1985, where it was *postulated* in order to justify "consistent histories" within a conventional probability interpretation, whereas my arguments of 1970 were derived from universal unitarity in an attempt to *resolve* the measurement problem – not to tolerate it. Ironically, it is precisely this consequence of universal unitarity that had led to the traditional prejudice that quantum theory does *not* apply to the macroscopic world.

still exists, but it "is not there" (somewhere) any more. Decoherence is in general a very drastic consequence of quantum dynamics, which requires (and allows) precise numerical calculations only for some mesoscopic systems, such as chiral molecules.[8] For example, some of the latter are found in chiral states on Earth, but in parity eigenstates in interstellar space, where the environment is weaker. In contrast, all objects that can be "seen" under normal conditions permanently scatter light, which is thus entangled with (and so may carry information about) the state of the object. If two positions can be distinguished by "just looking", the two quantum states of light must be orthogonal, and thus decohere the object. The position (rather than momentum) basis is here "preferred" by the locality of interactions in space.

The time asymmetry of the decoherence process (in *causing* entanglement) requires a low entropy initial condition for the wave function,[9] but without the concept of splitting observers ("many minds") as the other non-trivial consequence of global unitarity, decoherence would not be able to explain the observation of *individual* measurement outcomes. You cannot have one without the other if unitarity is generally valid. Decoherence has therefore occasionally been claimed to be insufficient to solve the quantum measurement problem. However, the subsequent splitting of observer states amounts for the latter to what Pauli once called the "creation of measurement results outside the laws of Nature", although it is now described as a dynamical consequence of global unitary dynamics *on the observer himself*. Instead of properly taking into account the environment and the role of the observer in a consistent quantum setting, that is, in a deeply entangled world, Pauli, Heisenberg and their disciples referred to an extra-physical observer and his "information" as a *deus ex machina*.

The experimental confirmation of decoherence as a smooth (though very fast) dynamical process has clearly demonstrated that the concept of entanglement does apply beyond microscopic systems. While this process must remain uncontrollable in order to be irreversible ("real" rather than "virtual"), it has many obvious and important consequences – including apparent quantum jumps and the classical appearance of the world (as consisting of particles *and* fields). So it explains why we seem to observe individual atoms as apparent particles in a Paul trap, or tracks in a Wilson chamber as apparent particle trajectories (both are correctly described in terms of narrow wave packets), and why one finds bound microscopic systems preferentially in their energy eigenstates.[7,10] It also allows us to understand the mysterious concept of "complementarity" simply by the different entanglement of microscopic objects with the environment, caused by means of different measurement instruments. This choice of "complementary measurement devices" is not available for systems, such as macroscopic ones, that are already strongly entangled with their unavoidable environment without being measured by a physicist. The basis "preferred" by this unavoidable environment defines a quasi-classical *configuration* space for such systems, which include even major parts (such as neural systems) of the thus partially classical observers. While *virtual* decoherence had always been known in the form of microscopic (reversible and often observable) entanglement, the unavoidable and irreversible consequence of the environment on macroscopic systems was overlooked for five decades, mainly because quantum mechanics was traditionally assumed *not* to apply beyond microscopic systems. Surprisingly, the apparently reversible classical mechanics does in quantum mechanics require the permanent (though mostly thermodynamically negligible) action of irreversible decoherence.

In order to illustrate the enormous number of new "worlds" that are permanently created according to decoherence (or would otherwise be permanently annihilated by a collapse mechanism), let me consider the example of a two-slit experiment. Measuring which slit the "particle" passes through would about double the number of worlds, but registration of the particle on the second screen causes a multiplication of worlds by a large factor that depends on the remaining coherence lengths for the positions of the decohered spots. (Everett "worlds" are not *exactly* separated, and thus cannot simply be counted; they may even form an overcomplete set.) This definition of branch worlds by their irreversible separation in configuration space means also that quantum computers do *not* simultaneously calculate in parallel worlds (as sometimes claimed) if they are to produce a coherent result that may then be used in "our" world, for example; "real" (rather than virtual) branches never recohere to form local superpositions again.

Most "particles" in the two-slit experiment do not even pass the slits, but are instead absorbed on (or reflected from) the first screen. This may correspond to a position measurement, too – regardless of whether its information is ever extracted. In order to cause decoherence, this "information" may even be thermalized (erased in the usual sense). In contrast, a "quantum eraser" requires a local superposition to be *restored*, that is, re-localized, rather than information to be destroyed, as its name may suggest. A similar formation of entanglement occurs in most accidental interactions between different quantum systems. For M such "measurement-like events" in the past history of the universe with, on average, N different outcomes, one would obtain the huge number of N^M now existing branches. Nonetheless, the global configuration space remains almost empty because of its huge dimension; the myriads of branching wave packets that have ever been created by *real* decoherence describe separate "worlds" for all reasonable times to come. Nobody can calculate such a global wave function, of course, but under appropriate (far from equilibrium) initial conditions for the universe, its unitary dynamics can be used consistently to justify (1) quasi-classical properties and behavior for all degrees of freedom that are "robust" under decoherence, (2) statistical methods (retarded probabilistic master equations) for most others,[9] and (3) individual wave functions for appropriately prepared microscopic systems. In the case of controllable non-local entanglement, this latter kind of preparation can even be applied at a distance – a phenomenon known as "quantum steering". These three dynamical applications are then also sufficient to construct measurement devices to begin with. No phenomenological concepts (such as particles, events, pointer positions, or even Alice and Bob) are required on a *fundamental* level.

The observation of radioactive decay represents another measurement of a continuous variable (namely, the decay time). Its precision cannot be better than the remaining coherence time (which is usually very much smaller than the half-life, and thus gives rise to apparent quantum jumps). This coherence time depends on the efficiency of the interaction of the decay fragments with their environment, and it would be further reduced by permanent registration of the (non-) decay. If an excited state decays only by emission of weakly interacting photons, however, decoherence may be relatively slow. In a cavity, one may then even observe coherence between different decay times, thus definitely excluding genuine quantum jumps ("events") in this case. There is no reason to believe that this would be different if the photon had travelled astronomical distances before a coherent state vector revival occurs.

Many leading physicists who are not happy any more with the Copenhagen interpretation nonetheless prefer to speculate about some novel kind of dynamics (an as yet unknown collapse mechanism) that would avoid the consequence of Many Worlds. This is at present no more than prejudice combined with wishful thinking, but it could in principle also solve the measurement problem in terms of an ontic (in this case partially localized) universal wave function without requiring Everett's multiple observers. One should keep in mind, though, that all as yet *observed* apparent deviations from unitarity, such as quantum jumps or measurements, can be well described (and have in several cases been confirmed experimentally) as smooth decoherence processes in accordance with a global Schrödinger equation. Therefore, if a genuine collapse mechanism did exist after all, it would presumably have to be *triggered* by decoherence, but it could then hardly have any observable consequences on its own.

For example, if one of two spatially separated but entangled microscopic systems (such as those forming a "Bell state") was measured, their total state would according to a unitary description become entangled with the apparatus, too, and thus also with the latter's environment. While this process leads to the formation of two dynamically autonomous branches, an observer at the location of the second system, say, becomes part of this entanglement (and therefore "splits") only when he receives a signal about the result. Before this happens, his state factors out, and he may be said not yet to *know* the result. If he also measured the second system (that at his own location in this case), the state of his memory must thereafter depend on the outcomes of both measurements, that is, it must have split twice unless there was an exact correlation between the results. Since the order of these two measurements does not matter, in general, this description includes delayed choice experiments. In contrast, a genuine collapse caused by the measurement would have to *affect* distant objects instantaneously (whatever that means relativistically) in order to avoid other weird consequences. This *would* then define the "spooky" part of the story.

However, an *apparent* ensemble of quasi-classical "worlds" is for all practical purposes ("FAPP") sufficiently defined by the autonomous branches of the wave function that arise from decoherence: a measurement cannot be undone in practice as soon as the global superposition cannot be re-localized in configuration space any more. Reasonable observer states can only evolve separately within the different branches. Neither can we as yet *exactly* define conscious observer systems in physical terms, nor does this mechanism completely explain Born's rule, since all members of the apparent ensemble remain part of one superposition (the "bird's perspective"). Observers in many branches would, in series of measurements, even describe frequencies of outcomes that are *not* in accord with Born's rule. What we still need, therefore, is a probabilistic characterization of the quasi-classical world in which "we" happen to live.

In all interpretations of quantum mechanics, Born's rule has to be *postulated* (in addition to the unitary dynamics) on empirical grounds in some form. In principle, this remains true in the Everett interpretation, too, but the situation is now partly solved by decoherence, as the *members* of an effective ensemble of potential physical "outcomes" (namely, the branches) have been sufficiently defined. (In contrast to the splitting of observers, this branching into autonomous "worlds" is not a fundamental concept; it is a consequence of global unitary dynamics.) According to their definition by robustness against further decoherence, there are no branches that contain Schrödinger cats or sugar molecules in parity eigenstates and the like. So their probability is zero. All we still have to postulate for the remaining branches are relative probability *weights*, which should not be affected by the imprecise and time-dependent definition of the branches. The only appropriate candidate for them is the squared norm (their *formal* measure of size), as it is additive and conserved under the unitary dynamics. It is thus not affected by any subsequent finer branching. (For example, further branching occurs during subsequent physical information processing, such as photon "measurements" on the retina, or by measurement-type events somewhere else in the universe.) Therefore, this property gives rise to individual probabilities for apparent collapse events, and thus to the concept of (apparent) "consistent histories". Everett regarded this dynamical argument, which is similar to the choice of phase space volume as a probability measure in classical statistical mechanics, as *proof* of Born's probabilities.[11] However, only *after explicitly postulating* them, does the density matrix (called a "mixed state") become justified as a conceptual tool.

By consistently using this global unitary description, all those much discussed "absurdities" of quantum theory can be explained. It is in fact precisely how they were all predicted – except that the chain of unitary interactions is usually cut off *ad hoc* by a collapse at the last relevant measurement in an experiment, where the corresponding decoherence defines a consistent position for the hypothetical Heisenberg cut. Therefore, all those "weird" quantum phenomena observed during the last 80 years can only have surprised those who had never seriously considered the possibility of a universal validity of unitarity. Absurdities, such as "interaction-free measurements", arise instead if one assumes the quasi-classical phenomena (such as apparent events) rather than the complete wave function as describing "reality". If the wave function itself represents reality, however, any "post-selected" component cannot, by itself, describe the previously documented past any more, which would have to be the case if this post-selection were no more than an increase of information.

So-called quantum teleportation is another example where one can easily show, using unitary dynamics, that nothing is ever "teleported" that, or whose deterministic predecessor, was not prepared *in advance* at its intended position in one or more components of an entangled initial wave function.[10] This confirms again that nonlocal wave functions cannot merely represent a bookkeeping device – even though a local observer *may assume* that an objective global collapse into a non-predictable outcome had already occurred (or that this outcome had been *created* in some other kind of "event") as a consequence of the first irreversible decoherence process after a measurement. It is precisely this possibility that justifies the usual pragmatic approach to quantum mechanics (including the Copenhagen interpretation or von Neumann's collapse during a measurement). However, if one assumed only local properties, such as quasi-classical measurement outcomes, to describe reality, one would indeed have to believe in teleportation and other kinds of spooky action at a distance. According to the Everett interpretation, the usual restriction of "our" quantum world to a tiny and permanently further collapsing *effective* wave function therefore represents no more than a pragmatic convention that reflects the observer's changing situation rather than a physical collapse process. Such a "collapse by convention" may even be assumed to apply instantaneously (superluminally), but it should be evident that a mere convention cannot be used for sending signals.

If the global state does indeed always obey unitary dynamics, the observed quantum indeterminism can clearly *not* reflect any objective dynamical law. In the Everett interpretation, it is in principle a "subjective" phenomenon that reflects the branching histories of all observers into many different versions ("many minds"). This may *explain* Heisenberg's original interpretation of quantum measurements as requiring "human" observers. This passive indeterminism is nonetheless essential for the *observed* dynamics of the world (that of its "relative state"). All measurement outcomes are thereby objectivized by the correlation between those versions of *different* observers (including Wigner's friend or Schrödinger's cat) who exist in the same Everett branch, and thus can communicate with one another. For all practical purposes, their entanglement with the apparatus after reading it, and with the environment, also justifies Bohr's interpretation of measurements (unlike Heisenberg's) in terms of classical outcomes that would be irreversibly and *objectively created* (in apparent events) by the macroscopic apparatus. This macroscopic entanglement (in addition to decoherence) explains the traditional concept of a "classical reality": only a documented phenomenon is a phenomenon (see also Footnote ** in Sect. 5). However, only if one misinterpreted the resulting global superposition as a genuine *statistical ensemble* consisting of the autonomous branches, would an observation of the outcome appear as a mere increase of information.

5. Quantum Field Theory

We have seen that quantum mechanics in terms of a universal wave function admits a consistent (even though novel kind of) description of Nature, but this does not yet bring the strange story of particles and waves to an end. Instead of *spatial* waves (fields) we were led to wave functions on a high-dimensional "configuration space" (a name that is justified only by its appearance as a space of *potential* classical states because of decoherence). For a universe consisting of N particles, this configuration space would possess $3N$ dimensions, but we may conclude from the arguments presented in Sect. 3 that for QED (quantum electrodynamics) it must be supplemented by the infinite-dimensional configuration space of the Maxwell fields (or their vector potentials in the canonical formalism). A product of wave functions for the amplitudes of all field modes in a cavity or in free space turned out to be sufficient to explain Planck's quanta by the number of nodes of these wave functions. The spontaneous occurrence of photons as apparent particles (in the form of clicking counters, for example) is then merely a consequence of the fast decoherence of the entangled state in the detector.

However, we know from the quantum theory of relativistic electrons that they, too, have to be described by a *quantized field* (that is, by a field functional) – a consequence that must remain true in the non-relativistic limit. There are no particles even *before* quantization any more. The relativistic generalization of a one-electron wave function is called the *Dirac field*, since it is usually studied as a function on spacetime. Dirac proposed it at a time when Schrödinger's wave function was mostly believed to define a spatial field for each electron, but the Dirac field can *not* be generalized to an *N*-electron field on a *4N*-dimensional "configuration spacetime", although this has occasionally been proposed. There is only one time parameter describing the dynamics for the total state. In the Schrödinger picture of QED, the Dirac field is used to define, by its configuration space and that of the Maxwell field, the space on which the corresponding time-dependent wave functionals live. According to the rules of canonical quantization, these wave functionals have to obey a generalized Schrödinger equation again (the Tomonaga equation).[12] Spin and other internal degrees of freedom thereby become part of the "classical" (not-yet-quantized) fields.

Non-relativistically, this consequence of QFT avoids a fundamental *N*-dependence of configuration spaces for different numbers *N* of "particles". Quite generally, it allows for a concept of "particle creation", such as by raising the number of nodes of the field functional (cf. Sect. 3). Relativistic covariance cannot and need not be manifest in this canonical formalism. For example, the canonical quantization of the Maxwell field leads consistently to a wave functional $\Psi\{\boldsymbol{A}(\boldsymbol{x});t\}$, with a vector field $\boldsymbol{A}$ defined at all space-points $\boldsymbol{x}$ on an arbitrary simultaneity t. Since Schrödinger had originally discovered his one-electron wave function by the same canonical quantization procedure (applied to a single mass point), the quantization of the Dirac field is for this purely historical reason also called a "second quantization". As explained in Sect. 4, though, the particle concept, and with it the first quantization, are no more than historical artifacts.[13]

Freeman Dyson's "equivalence" between using relativistic field functionals (Tomonaga) or field operators (Feynman)[14] is essentially based on the limited equivalence between the

Schrödinger and the Heisenberg picture. The Heisenberg picture would hardly be able even in principle to describe the hefty, steadily growing entanglement described by a time-dependent global wave functional. It is therefore mainly restricted to the quantization of *free* fields (coupled oscillators, which can easily be quantized algebraically). Since relativity is incompatible with absolute simultaneities, the relativistic generalization of the Schrödinger equation can only be given by the Tomonaga equation with its "many-fingered" concept of time (arbitrary simultaneities). Apparent particle lines in Feynman diagrams, on the other hand, are merely shorthand for free field modes (such as plane waves, with "particle momenta" representing their wave numbers).[3] These diagrams are used as intuitive tools to construct terms of a perturbation series in terms of integrals over products of such *field modes* and other factors – mainly for calculating scattering amplitudes. In this approximation, closed lines ("virtual particles") may represent local entanglement between quantum fields. Since high-energy physics is mostly restricted to scattering experiments, unitarity is in many textbooks quite insufficiently interpreted as describing the "conservation of probability" – thus neglecting its essential consequence for the quantum phases, which are needed to determine nonlocal superpositions that must arise in such a scattering process. This interpretation would *presume* a collapse.

The Hamiltonian form of the Dirac field equation is unusual because of its linearization in terms of particle momentum: the classical canonical momenta are not given by time derivatives of the position variables (velocities) any more. Nonetheless, the two occupation numbers 0 and 1 resulting from the *assumption* of anti-commuting field operators[§] are again

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§ Let me emphasize, though, that the origin of the Pauli principle, which is valid for fermions, does not seem to be entirely understood. While the individual components of the Dirac spinor also obey the Klein-Gordon equation, the latter's quantization as a field of coupled oscillators would again require *all* oscillator quantum numbers $n = 0,1,2,\ldots$. Anti-commuting field operators, which lead to anti-symmetric multi-particle wave functions, were postulated quite *ad hoc* by Jordan and Wigner, and initially appeared artificial even to Dirac. Interpreted rigorously, their underlying configuration space (defining a Hilbert space basis again) would consist of a spatial continuum of coupled bits ("empty" or "occupied") rather than a continuum of coupled oscillators. The n-th excited state of this bit continuum (that is, n occupied positions) represents n *identical* point-like "objects". Because of the dynamical coupling between bit-neighbors, these objects can move, but only *after* their quantization, which leads to entangled superpositions of different occupied space points, may they give rise to propagating waves. In order to be compatible with this bit continuum, the coefficients of these superpositions ("multi-fermion wave functions") must vanish whenever two of their arguments coincide. This can quite generally be achieved by assuming them to be antisymmetric under permutations of any two arguments. No field algebra is explicitly required for this argument (although it could then be consistently defined). In this picture, single-fermion wave functions would represent genuine quantum states (quantum superpositions) rather than wave modes as for bosons. In contrast, coupled oscillators defining a free boson field propagate as spatial waves, and thus obey a *classical* superposition principle (in space rather than in their configuration space) in addition to the quantum superposition principle that is realized for them by the field functionals. This difference would be particularly dramatic in Bohm's theory, where one often meets disagreement on whether its trajectories have to include photons as particles or as a time-dependent vector potential (a classical field). However, these pre-quantization concepts need not possess any physical meaning by themselves. Moreover, such a fundamental distinction between bosons and fermions may be problematic for *composite* "particles" (dressed fields).

interpreted as "particle" numbers because of their consequences in the quasi-classical world. Field modes "occupied" once in this sense and their superpositions define again "single-particle wave functions". In contrast to the case of photons, however, one never observes superpositions (wave functionals) of *different* electron numbers. This has traditionally been regarded as a fundamental restriction of the superposition principle (an axiomatic "superselection rule"), but it may be understood as another consequence of decoherence: for charged particles, their Coulomb field assumes the role of an environment.[15]

In QFT, the formulation that one particle is in a quantum state described by the spatial wave function ψ_1, and a second one in ψ_2, is thus replaced by the statement that two *field modes*, ψ_1 and ψ_2, are both in their first excited quantum state ("occupied once"). A permutation of the two modes does not change this statement that is based on a logical "and", so there is only *one* state to be counted statistically. This eliminates Gibbs' paradox in a very natural way. (Schrödinger seems to have used a similar argument in favor of waves instead of particles even before he explicitly formulated his wave equation.[16])

It would similarly be inappropriate to claim that *wave functions* can be directly observed in Bose-Einstein condensates (as is often done). What one observes in this case are again the (now many times "occupied") three-dimensional boson *field modes* – including massive bosons, which are traditionally regarded as particles. Instead of the *free* field modes used for photons, however, interacting bosons are then more appropriately described in terms of self-consistent field modes in analogy to the self-consistent Hartree-Fock single-fermion wave functions. Both methods neglect any "particle" entanglement, and can therefore at most represent approximations for ground states. They lead to what is regarded as an effective non-linear "single-particle wave equation" – for bosons called the Gross-Pitaevskii equation.** In

** At higher temperatures, "many-particle" systems (that is, multiple quantum field exitations) may behave approximately like a gas of classical particles undergoing stochastic collisions because of the mutual decoherence of the field modes into apparent ensembles of narrow spatial wave packets.[28] This consequence perfectly justifies Boltzmann's *Stosszahlansatz* – but *not* any quasi-deterministic particle trajectories. The concept of trajectories would approximately apply only to heavy objects that suffer mainly "pure" decoherence (with negligible recoil). "Open" quantum systems are generally described by similar phenomenological (Lindblad-type) master equations that are usually *postulated* rather than being derived from realistic assumptions for a quantum environment, and often misunderstood as representing fundamental deviations from unitary quantum mechanics. In order to be regarded as "macroscopic" in the sense of not being part of a thermal distribution, quasi-classical (decohered) variables have furthermore to be redundantly documented in the rest of the universe (see under "fork of causality", "consistency of documents", or "overdetermination of the past" in the first Ref. 7 – for example in Footnote 1 on its page 18). Dynamically conserved "information" *about* such systems may nonetheless be dynamically exchanged between microscopic and macroscopic variables, that is, between negentropy and macroscopic information. – In the theory of "quantum Darwinism",[29] these *classical* thermodynamic arguments are combined (and perhaps a bit confused) with the quantum concept of decoherence, which represents spreading physical entanglement, but not necessarily any spreading of (usable) information into the environment. Transfer of (necessarily spite of this effective non-linearity, the quantum states proper are, of course, still described by the linear Schrödinger equation – relativistically always in the sense of Tomonaga.[12]

As already mentioned in Sect. 3, photon number eigenfunctions $\psi^{(n)}(q)$ in the configuration space of wave amplitudes q – to be distinguished from their three-dimensional field modes ("single-photon wave functions", which are fixed modes in a cavity in this case) – have recently been observed and confirmed to exist for various values of the "particle number" n by means of their Wigner functions.[4] For pure states, Wigner functions are defined as partial Fourier transforms of the dyadic products $\psi^{(n)}(q)\psi^{(n)*}(q')$, and thus equivalent to the wave functions $\psi^{(n)}(q)$ themselves (except for a total phase). The variable q is here the amplitude of the given field mode rather than some spatial position as in single-particle quantum mechanics. The two-dimensional Wigner functions on their apparent phase space q,p were made visible in this experiment, and so allow one to clearly recognize the n nodes of the wave functions $\psi^{(n)}(q)$ (forming circles in phase space). Creation and annihilation operators are defined to change the number of these nodes. Since these operators occur dynamically only in the Schrödinger equation, they describe *smooth* physical processes (time-dependent wave functionals), while creation "events" are either meant just conceptually, or would require a fast decoherence process. The *physical* nature of the field functionals is also confirmed by their ability to participate in the general nonlocal entanglement and, in this way, contribute to the observable decoherence of sources of radiation without having to affect any absorbing matter as a further environment.

For relativistic reasons, *all* known elementary physical objects are described as quantum fields (although they are usually called "elementary particles"). The contrast between the first order in time of the Schrödinger equation and the second order of classical field equations with their negative frequencies opens the door to the concept of "anti-bosons". (For fermions this relation assumes a different form – depending on the starting point before quantization, as indicated in Footnote §.) Because of the universality of the concept of quantum fields, one may also expect a "theory of everything" to exist in the form of a unified quantum field theory. At present, though, the assumption that the fundamental arena for the universal

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physical) information must always cause decoherence in the source, but the opposite is *not* true: even an environment in thermal equilibrium may allow the formation of further entanglement with a "system under consideration". Documents which define *humanistic history* – including the history of science – obviously require even more specific correlations (which would define a specific "context").

wave function be given by the configuration space of some fundamental field(s) is no more than the most plausible attempt. On the other hand, the general framework of Schrödinger's wave function(al) or Dirac's superposition principle as a universal concept for quantum states which obey unitary dynamics has always been confirmed, while attempts to derive this framework from some deeper ("hidden") level have failed and are strongly restricted by various no-go theorems. Therefore, an epistemic interpretation of quantum states seems to be ruled out.

Among boson fields, gauge fields play a surprisingly physical role, since gauge transformations appear locally as unphysical redundancies. Their physical role is facilitated by their dynamical entanglement, which thus reveals that the redundancy holds only classically; gauge variables then appear as purely relational quantities.[17] An important question after quantization is whether gauge symmetries can be broken by a real or apparent collapse.

Unfortunately, *interacting* fields require the entanglement of such an enormous number of fundamental degrees of freedom – traditionally interpreted as "quantum fluctuations" even in time-independent states – that they cannot even approximately be treated beyond a questionable (though within its applicability very successful) perturbation theory in terms of *free* effective fields. This limitation in practice of QFT to quantum oscillators may also explain the prevailing preference for the Heisenberg picture. Instead of consistently applying the established concepts from quantum mechanics (general superpositions) to the new variables (field amplitudes) in the form of time-dependent field functionals, various semi-phenomenological concepts are therefore used for specific purposes – mostly for calculating scattering amplitudes between phenomenologically chosen "objects" that are treated as being asymptotically free (which can be approximately true only in exceptional situations, such as high energy laboratory experiments). The much studied *S*-matrix can thus only be of limited value, since unitary dynamics is a continuous process rather than a succession of scattering events. It may reflect properties of its "objects", but it can clearly not explain them.

Stable local entanglement between different fields may be regarded as their "dressing" (similar to the entanglement between proton and electron in the bound hydrogen atom – cf. Sect. 4), while chaotic nonlocal entanglement must describe decoherence, and thus lead to the appearance of scattering as a probabilistic rather than a unitary process. Only for individual field modes, as in cavity QED, may one explicitly study their entanglement, for example that with individual atoms.

Similar semi-phenomenological methods as in QFT are also used in condensed matter physics, even when its objects of interest are non-relativistically regarded as *N*-particle systems. They may nonetheless give rise to effective phonon fields or various kinds of "quasi-particles". The wave function for the lattice ions and their electrons, for example, is here regarded as fundamental, while the phonon field functional "emerges" – similar to Goldstone bosons in QFT. Symmetry-breaking effective ground states (such as lattices with fixed positions and orientations) and their corresponding "Fock spaces" can be understood as representing Everett branches that have become autonomous by the decoherence of their superpositions into wave packets during a condensation process.[18] Some such "Fock vacua" are characterized by the number of certain particles (such as electrons in a metal) that form a *stable entanglement* in this ground state. Most familiar are pair correlations in the BCS model of superconductivity. A similar model in QFT led to the prediction of the Higgs "particle". However, only in situations described by an effective Hamiltonian that gives rise to an energy gap (defining an effective mass) can the lowest excited states approximately avoid further decoherence within their corresponding Fock space under normal conditions and low temperatures, and thus exhibit the usual phenomena of "single particle" quantum mechanics.

The BCS (pair correlation) model is also useful for understanding the origin of Hawking and Unruh radiation,[19] which are often misinterpreted as representing vacuum fluctuations rather than entanglement. Since only exceptional field modes of a given space volume would obey boundary conditions also for any given subvolume, not even the total ground state factorizes into *local* subvacua. The Hilbert space Hamiltonian, therefore, depends not only on the differential operators, but also on the chosen boundary conditions (which define its eigenstates), while complementary subvolumes are entangled in almost all pure *total* states of QFT. For parallel *physical* boundaries, which require an infinite energy renormalization corresponding to the zero-point energies of all thereby excluded field modes, this leads to the Casimir effect as a measurable (finite) dependence on distance between the plates. In the absence of physical boundaries, the entanglement may be regarded as a *static* mutual decoherence of open subvolumes in a pure total state. Non-inertial detectors, for example, define real or apparent spacetime horizons as formal boundaries for the modes to which they couple, and thus register an (improper) thermal mixture representing Hawking or Unruh radiation in the inertial vacuum (which, in contrast, extends beyond these horizons). The presence of "particles" (field excitations) is here a matter of spacetime perspective, based on the choice of non-inertial reference frames (such as Rindler frames) that are used to define "plane" waves as relevant field modes, while general quantum *states*, such as different "physical vacua", are *objectively* defined by their physical (for example, cosmological) boundary conditions – and thus represent "real" states in spite of their ambiguous interpretation in terms of particles.

In *microscopic* many-particle systems, for example in small molecules or atomic nuclei, spontaneous symmetry breaking may even lead to energy eigenstates for collective motions (such as rotations or vibrations). Since electrically neutral microscopic objects can often be assumed to be isolated from their environments, asymmetric "model ground states" (deformed nuclei or asymmetric, such as chiral, molecular configurations) are degenerate and thus lead to energy bands or multiplets by means of different superpositions of all their degenerate orientations or chiralities.[20] The corresponding collective degrees of freedom are often classically visualized as describing slow ("adiabatic") motion, although this would in turn require time-dependent superpositions of *different* energy eigenstates. The quantum mechanical justification of such time-dependent states, which are found for *macroscopic* objects, had to await the discovery of decoherence (here of the energy eigenstates). Since all particles in a collective superposition of different orientations are strongly entangled with one another, energy eigenstates are analogous to the bird's perspective of a quantum world, while an external observer of such an eigenstate assumes the role of a "real bird". In contrast, the whole quantum world must *contain* and thus be entangled with its observer, who thus gives rise to "many minds" with their asymmetric frog's perspectives (broken symmetries).[18] In accordance with this picture, individual particles that are parts of collective rotational superpositions feel in first approximation only a *fixed* deformed potential (analogous to observing a definite measurement outcome), as can be seen from their single-particle spectra – for example Nielson states in deformed nuclei as a variant of the nuclear shell model. (This observation was the starting point for the many-minds interpretation.) In this sense, collective superpositions imitate a "multiverse" consisting of different orientations, but such quantum cosmological analogies seem to have delayed the acceptance of the concept of decoherence for a decade, until its "naïve" interpretation by means of the pragmatically justified reduced density matrix formalism became popular and made it acceptable to many practicing quantum physicists. In the case of a *global* symmetry, collective variables bear some similarity to gauge variables.

On a very elementary level, semi-phenomenological methods were already used for the hydrogen molecule by separately quantizing its "effective" degrees of freedom (center of mass motion, vibration, rotation and two independent electrons in the Coulomb field of adiabatically moving nuclei) rather than treating it exactly as an entangled four-body problem.

Chiral molecules can at very low energies effectively be described as two-state systems, while an analogous explanation may conceivably await discovery for *all* kinds of qubits.

In QFT, the successful phenomenology of apparently fundamental fields ("elementary particles"), such as described by the Standard Model, has to be expected to form the major touchstone for any fundamental theory of the future. This may be true even though quantum chromodynamics seems to be already too complex for us to derive nuclear physics phenomena without auxiliary assumptions. This Standard Model is essentially based on linear representations of some abstract symmetry groups, whose meaning is not yet understood. The physical importance of linear representations of groups for isolated systems is just another consequence of the superposition principle. At present, however, the Standard Model does not seem to offer any convincing hints for the nature of the elusive fundamental theory.

All one may thus dare to predict is that the fundamental Hilbert space must possess a local *basis* (such as the configuration space of spatial fields of local properties and/or point-like objects) in order to allow for a definition of dynamical locality or "relativistic causality". In contrast to popular concepts of *mono*-causality, however, classical reality is multi-causal: in order to determine the fields at some spacetime point in classical field theory, one has to know them on a complete slice through its (past or future) light cone. Only since the causal connection between two *events* may then be difficult to check, did Einstein postulate the travel of "signals", which may be characterized by some identifyable structure, rather than general causal influences, to be limited by the speed of light. Although *quantum superpositions* of such fields are kinematically nonlocal, and thus able to violate Bell's inequality, the *dynamical* locality defined for their basis remains valid and important for them (including interactions that describe measurements and decoherence). This relativistic causality may even prevent the formation of black hole horizons.[21] While nonlocal phase relations defining superpositions are essential for the precise value of von Neumann's conserved global ensemble entropy (zero for pure states), the dynamical transformation of information about local systems into that about nonlocal correlations or entanglement describes the increase of "physical entropy", since the latter is defined as additive and thus neglects nonlocal correlations for being thermodynamically "irrelevant".[9]

This search for the Hilbert space basis of a fundamental theory has nothing to do with that for "hidden variables", which are to explain quantum indeterminism and the wave function themselves. All novel theories that are solely based on mathematical arguments, however, have to be regarded as speculative until empirically confirmed – and even as incomplete as long as there is no general consensus about the correct interpretation of their quantization. Many quantum field theorists and mathematical physicists seem to regard their semi-phenomenological models, combined with certain methods of calculation and applied to classical field or particle concepts, as *the* quantum field theory proper. Indeed, why should one expect a consistent theory if there is no microscopic reality to be described – as assumed in the still popular Copenhagen interpretation and its variants? Therefore, most textbooks of QFT do not even *attempt* to present a conceptually consistent and universally applicable theory.

Our conclusion that the observed particle aspect is merely the consequence of fast decoherence processes in the detecting media does not seem to be of particular interest to many high-energy physicists, although such phenomena in their detectors are an essential part of their experiments. Some of them call the enigmatic objects of their research "wavicles", as they cannot make up their mind between particles and waves. This indifferent language represents another example of Wigner's "Balkanization of physics" (or "many words instead of many worlds" according to Tegmark). The wave-particle "dualism" is usually still understood with respect to *spatial* waves rather than wave functions in configuration space, although the former should by now be known to be quite insufficient for quantum theory.

6. Quantum Gravity and Quantum Cosmology

I cannot finish a presentation of universal quantum theory without having mentioned quantum gravity. In their linear approximation, Einstein's field equations for the metric tensor define separately oscillating spatial tensor modes, that after quantization give rise again to energy quanta $h\nu$ ("gravitons") – cf. Sect. 3. However, for consistency the full theory must also be quantized. Its dynamical variables must then appear among the arguments of a universal wave function, and thus be entangled with all others – in a very important way, as it turns out.[22]

The Hamiltonian formalism of Einstein's nonlinear field equations, required for a "canonical" quantization (here as an effective theory that cannot be valid at very high energies), was brought into a very plausible form by Arnowitt, Deser and Misner in 1962. They demonstrated that the configuration space of gravity can be understood as consisting of the spatial geometries of all possible three-dimensional space-like hypersurfaces in spacetime. These hypersurfaces define arbitrary simultaneities that may form various foliations of spacetime, which may then be parametrized by a time coordinate t. This Hamiltonian form of the theory is therefore also called "geometrodynamics". Its canonical quantization leads to a (somewhat ambiguously defined) Schrödinger equation in the sense of Tomonaga for the wave functional on all these geometries – known as the *Wheeler-DeWitt equation*. This is another example of the fact that the Hamiltonian form of a theory is not in conflict with its relativistic nature.

In contrast to the normal Schrödinger equation, the WDW equation remarkably assumes the form $H\Psi = 0$. It can also be understood as a constraint, while the Schrödinger equation itself then becomes trivial: $\partial\Psi/\partial t = 0$. The reason is that there is no classical spacetime any more to be foliated. (Each foliation of spacetime would correspond to a *trajectory* through this configuration space – in conflict with quantum theory). However, the spatial metric that occurs (besides matter variables) as an argument of the wave functional Ψ would determine all proper times (physically meaningful times) along time-like curves which connect it classically, that is, according to the Einstein equations, with any other given spatial geometry – regardless of the choice of a foliation. Therefore, in spite of its formal timelessness, the Wheeler-DeWitt equation *does* define a physical time dependence by means of the entanglement between all its variables – similar to the entanglement $\psi(u,q)$ between a clock variable u and other variables q in quantum mechanical description. Therefore, the *formal* timelessness of the WDW equation is a genuine quantum property that reflects the absence of trajectories. Classical spacetimes correspond to trajectories that can be parametrizied by a coordinate time t (albeit invariantly reparametrizable by monotoneous functions $t'(t)$). *Physical* time is in general many-fingered, that is, it depends on the local progression of the space-like hypersurfaces independently at any space point. In the case of an exactly homogenous and isotropic Friedmann cosmology, it may conveniently be represented by *one* single "finger": the expansion parameter a. If the wave function is regarded as a probability amplitude, it now defines probabilities *for* physical time; it is not a function *of* (some external) time any more.

It is further remarkable in this connection that, for Friedmann type universes, the Hamiltonian constraint $H\Psi = 0$ assumes a hyperbolic form in its infinite-dimensional (gauge-free) configuration space – again with a or its logarithm defining a time-like variable. This property is physically very important, since it allows for a global "initial" value problem for the wave functional – for example at $a \to 0$.[23] For increasing a, its solution may form a superposition of wave packets that "move" through this configuration space as a function of a. A drastic asymmetry of Ψ with respect to a reversal of a (an "intrinsic" arrow of time) might then be derivable even from symmetric boundary conditions (such as the usual integrability condition in a) because of the asymmetry of the Hamiltonian under this reversal.

Claus Kiefer could furthermore derive the time-dependent Schrödinger (Tomonaga) equation for the matter wave function under a short wave length approximation for the geometric degrees of freedom. It corresponds to a Born-Oppenheimer approximation with respect to the inverse Planck mass (see Kiefer's Ch. 4 in Joos et al. of Ref. 7, or his Sect. 5.4 of Ref. 22). This Hamiltonian form emphasizes the fact that the Wheeler-DeWitt equation can only describe a whole Everett multiverse, since each trajectory in the configuration space of spatial geometries would define a (possibly different) classical spacetime. Wave packets for spatial geometry approximately propagating along such trajectories are decohered from one another by the matter variables (which thereby serve as an "environment"). This is analogous to the decoherence of atomic nuclei in large molecules by collisions with external particles – the reason why they appear to move on quasi-classical trajectories according to the frog's perspective of a human observer. In cosmology, decoherence (that is, uncontrollable entanglement rather than the often mentioned "quantum fluctuations") is also important for the origin of "classical" structure in the early universe during the onset of inflation.[24]

If one also allowed for "landscapes" (Tegmark's Level 2 of multiverses[25]), which are assumed to exist in several hypothetical cosmologies that lead to a drastically inhomogeneous universes on the very large scale, the "selection" (by chance – not by free will) of a subjective observer with his epistemologically important frog's perspective (cf. Sect. 4) may be roughly characterized by a hierarchy of five not necessarily independent steps: (1) the selection (in the sense of Level 3, that is, Everett – usually regarded as a quantum measurement) of an individual landscape from their superposition that must then be part of a global quantum state, (2) the selection of a particular region in this three or higher dimensional landscape (a causally separate "world" that may even be characterized by specific values of certain "constants of nature" – Level 2), (3) the selection of a quasi-classical spacetime from the Wheeler-DeWitt wave function as indicated above (Level 3 again), (4) the selection of one individual complex organism from all those that may exist in this "world", including some "moment of awareness" for it (giving rise to an approximate localization of this observer in space and time: a subjective "here-and-now" – thus including Level 1), and (5) the selection of one of his/her/its "versions" that must have been created by further Everett branching based on the decoherence of matter variables according to Sect. 4 (Level 3). Therefore, every conceivable subjective observer who is part of the universe represents an extreme "individualization" (multiple localization) *in* the real quantum universe, and hence *of* his observed "world" (his frog's perspective). This individualization seems to be required in order to define IISs (integrated-information systems), IGUSs (information gaining and utilizing systems), or however you call systems that may potentially form the physical basis for conscious beings, and, therefore, for an *observable* universe. New *physical* laws may not be required for this purpose.

Each of these steps may create its own unpredictable initial conditions characterizing the further evolution of the resulting individual worlds. Most properties characterizing our observed one can thus not be derived from any physical theory; they would have to be empirically determined as part of an answer to the question: *Where* do *we* happen to live in objective "configuration" space? This unpredictability, including that of certain "constants of nature", and complained about by some mathematical physicists and cosmologists, is by no means specific for a multiverse (as some critics argue). It would similarly apply to any kind of stochastic dynamics (such as in collapse theories), or whenever statistical fluctuations are relevant during the early cosmic evolution. Only step 4 can *not* be objectivized in the usual sense, namely with respect to different observers in the same quasi-classical world. Some of these steps may require an application of the weak anthropic principle in some sense (although I would not recommend to *rely* on it for the future by playing "Russian quantum roulette"!). Although each *individual* unpredictable outcome must be quite improbable, the observed ones should not be "unusually improbable". This is still a strong condition, which may even suffice to explain frequencies of measurement results according to Born's rule (Sect. 4). Entropy may *decrease* during most of these steps (depending on its precise definition).[6,9,26]

Let me add for completeness that Tegmark's Level 1 and 2 multiverses are classical concepts, and thus unrelated to Everett's branches, as they merely refer to separate regions in conventional space rather than branches in "configuration" space. It appears somewhat pretentious to speak of "parallel worlds" or a "multiverse" in this case; these names were originally invented for Everett branches, and are here simply misused. The reason may be that many cosmologists had never accepted the role of entangled superpositions as part of quantum reality, and therefore prefer to tacitly replace them by statistical correlations characterizing a collapse mechanism, for example. In this case, different outcomes could be realized only at different locations in a sufficiently large three-dimensional universe, while different Everett "worlds" exist even for a finite (closed) universe that represents a traditional big bang. (However, even different kinds and sizes of universes may formally exist in one superposition if the superposition principle is valid for them.)

While landscapes with regions of different properties would be quite plausible in a spatially unbounded or very large universe without making use of Everett (similar to locally varying order parameters resulting from symmetry breaking phase transitions[18]), almost *iden-*

tical local situations occurring by chance somewhere in an infinite quasi-homogeneous world (Level 1) may be regarded as something between trivial (entirely irrelevant for us) and ill-defined. Although the double exponentials which are needed to describe the expected distances from such statistical Doppelgängers can easily be *formulated*, an extrapolation of local properties (such as an approximately flat quasi-classical space) from the observable universe with its size of 10^{10} *ly* to something like $10^{10^{100}}$ *ly* appears at least risky. Statistical estimates of probabilities would in any case apply only to chance fluctuations (such as "Boltzmann brains"), but not to situations resulting from evolution. Their probabilities, if calculated by means of some *physical* (that is, additive) entropy, would completely neglect the existence of "consistent documents" (often regarded as an "overdetermination of the past" – see Footnote ** above and Sect. 3.5 of the first Ref. 9), while *unstructured* initial conditions (such as the initial homogeneity of a gravitating universe) represent even lower entropy values – in spite of their "plausibility".

The role of Tegmark's (as yet unmentioned) Level 4 universes is even entirely questionable, since mathematics, while providing extremely useful conceptual tools for physics because of its analytical (tautological) nature and, therefore, the undeniable *formal truth* of its theorems, cannot by itself warrant the *applicability* of specific formal concepts to the empirical world. Only if, and insofar as, such kinematical concepts have been empirically verified to be consistently applicable in a certain context, can we consider them as candidates for a description of "reality". (This seems to be a point that many mathematicians working in theoretical physics and cosmology have problems to understand, since they are used to define their concepts for convenience.) Different mathematical frameworks can therefore not be regarded as indicating the existence of corresponding different physical "worlds" or different parts of one world. While Everett's "many worlds" (just as all scientific cosmology) result from hypothetical extrapolation of the observed world by means of empirical laws, there are no reasons supporting the physical existence of Level 4 worlds. The *mathematical* concept of "existence", for example, means no more than the absence of logical inconsistencies, that is, a necessary (hence important[27]) but not a sufficient condition for being "realized" in Nature.

7. Conclusions

These remarks about quantum gravity and quantum cosmology may bring the strange story of particles and waves to a preliminary end. While the particle concept has been recognized as a mere delusion, the observed wave aspects of microscopic objects can be understood only as part of a universal wave function in a very-high-dimensional (if not infinite-dimensional) "configuration" space. If this wave function describes fundamental reality, no *local* properties can generally "exist". Only if one insists that reality *must* be defined in space and time, would this whole concept (which essentially means a unique and consistent description of Nature) have to be abandoned. The prejudice that reality has to be local seems indeed to represent the major hurdle for most physicists to correctly understand quantum theory.

The *observable* quantum world that defines our frog's perspective (that is, the relative state with respect to an individual "single mind") is thus no more than a tiny component of this global wave function. The latter, representing the "bird's view", may be regarded as the true *hidden* reality behind the phenomena, since its existence is required (and facilitated) from our point of view only for reasons of dynamical consistency. This is similar to the existence of objective spacetime in classical GR in spite of the absence of an absolute reference frame. Decoherence into dynamically arising new autonomous sub-branches of the wave function then mimics a collapse process, and in this way solves the measurement problem in terms of many (branching) minds – cf. Sect. 4. The full Wheeler-DeWitt wave function, for example, seems to be meaningful only from the bird's perspective, since it describes a superposition of many different spacetimes (which is permanently decohered further by means of "environmental" matter variables).

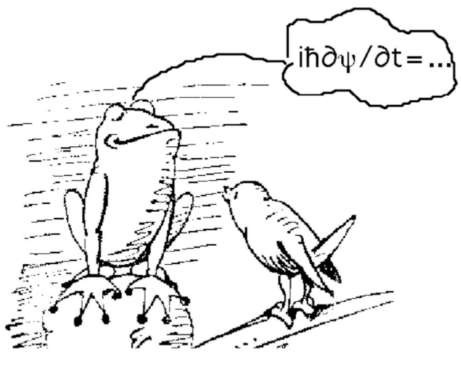

Fig. 2: A frog's bird's perspective

Matrix mechanics with its formal concept of "observables" thus turns out to be only an effective probabilistic description in terms of not consistently applicable (hence mutually "complementary") particle or other traditional concepts, which may in certain situations approximately apply to the observed world (our branch). Many physicists are still busy constructing absurdities, paradoxes, or no-go theorems in terms of such traditional concepts in order to demonstrate the "weirdness" of quantum theory. This includes black holes that disappear as a consequence of QFT (by Hawking radiation), but which are nonetheless often represented by classical Penrose diagrams (giving rise to the "information loss paradox").[21] Even Alice and Bob are classical concepts that have quantum mechanically to be justified by means of decoherence, caused locally by an uncontrollable environment. "Quantum Bayesianism", presently much *en vogue*, even replaces the whole physical world by a black box, representing an abstract concept of "information" about inconsistent classical concepts, and assumed to be available to some vaguely defined "agents" rather than to observers who may be consistent parts of the physical world to be described. In contrast to Everett's Many Worlds, for example, such a "non-theory" can never be falsified (it is "not even wrong").

While effective concepts like particles and spatial fields remain important for our every-day life, including that in physics laboratories, their limited validity must deeply affect a consistent world model (cosmology, in particular). It is always amazing to observe how the love affair of mathematical physicists and general relativists with their various classical field theories often prevents them from accepting, or even from sufficiently understanding, nonlocal quantum states that are well-known from quantum mechanics. Some of them are even trying to "explain" the fundamental quantum entanglement by means of speculative "worm holes" in space – apparently an attempt to save their belief in local reality. Quantum affects are then often belittled as mere "anomalies" in classical field theories.

We have to accept, however, that the precise structure of a local Hilbert space *basis*, which is often assumed to be given by the configuration space of some fundamental fields, remains elusive. Because of the unavoidable entanglement of all variables, one cannot expect the *effective* quantum fields, which describe apparent "elementary particles", to be related to these elusive fundamental variables in a simple way. This conclusion puts in doubt much of the traditional approach to QFT, which is based on concepts of renormalization and "dressing". There are indeed excellent arguments why even emergent ("effective") or quasi-classical fields may be mathematically elegant – thus giving rise to the impression of their fundamental nature. Novel mathematical concepts might nonetheless be required for finding the elusive ultimate theory, but their applicability to physics would have to be demonstrated empirically, and can thus never be confirmed to be *exactly* valid. This may severely limit the physical value of many "abstract" (non-intuitive) mathematical theorems. Just think of Einstein's words "*Insofern sich die Sätze der Mathematik auf die Wirklichkeit beziehen, sind sie nicht sicher, und insofern sie sicher sind, beziehen sie sich nicht auf die Wirklichkeit*", or Feynman's remark regarding early attempts to quantize gravity:[3] "Don't be so rigorous or you will not succeed." Fundamental physical laws and concepts have so far mostly turned out to be mathematically relatively simple, while their applications may be highly complex. This fact may explain why mathematicians have dominated theoretical physics mostly *after* completion of a new fundamental theory (such as Newton's and even more so Einstein's – but *not* yet for quantum theory!), or at times of stagnation, when mere reformulations or unconfirmed formal speculations (such as strings at the time of this writing) are often celebrated as new physics.

References:

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[1] E. Segrè, *From Falling Bodies to Radio Waves – Classical Physicists and their Discoveries* (Freeman, NY 1984)

[2] M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw-Hill 1966)

[3] H. D. Zeh, Feynman's interpretation of quantum theory, Eur. Phys. J. **H36**, 147 (2011) – arxiv:0804.3348 (2008)

[4] S. Deléglise et al., Reconstruction of non-classical cavity field states with snapshots of their decoherence, Nature **455**, 510 (2008) – cf. also Fig. 3.14 in E. Joos et al. from Ref. 7

[5] I. Pilkowski, M. Zych, F. Costa, and C. Bruckner, Universal decoherence due to gravitational time dilation, Nature Phys. **11**, 668 (2015); H. D. Zeh, Comment on decoherence by time dilation, arxiv:1510.02239 (2015)

[6] H. D. Zeh, The role of the observer in the Everett interpretation, NeuroQuantology **11**, 97 (2013) – arxiv:1211.0196 (2012)

[7] H. D. Zeh, On the interpretation of measurement in quantum theory, Found. Phys. **1**, 69 (1970); E. Joos and H. D. Zeh, The emergence of classical properties through interaction with the environment, Z. Phys. **B59**, 223 (1985); W. H. Zurek, Decoherence and the transition from quantum to classical, Physics Today **44** (Oct), 36 (1991); W. H. Zurek, Preferred states, predictability, classicality, and the environment-induced decoherence, Progr. Theor. Phys. **89**, 281 (1993); H. D. Zeh, What is achieved by decoherence? arxiv:quant-ph/9610014 (1996); L. Diósi and C. Kiefer, Robustness and Diffusion of Pointer States, Phys. Rev. Lett. **85**, 3552 (2000); E. Joos et al., *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer 2003), Chs. 1-4; M. Schlosshauer, *Decoherence and the quantum-to-classical transition* (Springer 2007)

[8] J. Trost and K. Hornberger, Hund's paradox and the collisional stabilization of chiral molecules, Phys. Rev. Lett. **103**, 023202 (2009)

[9] H. D. Zeh, *The Physical Basis of the Direction of Time,* 5th edn. (Springer, 2007); H. D. Zeh, Open questions regarding the arrow of time, arxiv:0908.3780 (2009)

[10] H. D. Zeh, There are no quantum jumps, nor are there particles, Phys. Lett. **A172**, 189 (1993); H. D. Zeh, Quantum discreteness is an illusion, Found. Phys. **40**, 1476 (2010); www.zeh-hd.de/Teleport.html

[11] P. Byrne, *The Many Worlds of Hugh Everett III* (Oxford UP, 2010), Ch. 27

[12] S. Tomonaga, On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields, Progr. Theor. Phys. **1**, 27 (1946); E. C. G. Stückelberg, Die Wechselwirkungskräfte in der Elektrodynamik und in der Feldtheorie der Kernkräfte, Helv. Phys. Acta **11**, 225 (1938)

[13] H. D. Zeh, There is no 'first' quantization, Phys. Lett. **A309**, 329 (2003)

[14] F. J. Dyson, The Radiation Theories of Tomonaga, Schwinger and Feynman, Phys. Rev. **75**, 486 (1949)

[15] D. Giulini, C. Kiefer, and H. D. Zeh, Symmetries, superselection rules, and decoherence, Phys. Lett. **A199**, 291 (1995)

[16] C. Joas and C. Lehner, The classical roots of wave mechanics: Schrödinger's transformations of the optical-mechanical analogy, Stud. Hist. Phil. Mod. Phys. **40**, 338 (2009)

[17] C. Rovelli, Why gauge? Found. Phys. **44**, 91 (2014)

[18] H. D. Zeh, Symmetry-breaking vacuum and state vector reduction, Found. Phys. **5**, 371 (1975)

[19] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space,* (Cambridge UP, 1983)

[20] R. E. Peierls and J. Joccoz, The Collective Model of Nuclear Motion, Proc. Phys. Soc. London **A70**, 381 (1957); H. D. Zeh, Symmetrieverletzende Modellzustände und kollektive Bewegungen, Nucl. Phys. **202**, 38 (1967)

[21] www.zeh-hd.de/nonlocality; H. D. Zeh, The nature and origin of time-asymmetric spacetime structures, In: *Springer Handbook of Spacetime Physics*, A. Ashtekar and V. Petkov, edts. (Springer, 2013), pp. 185-296 – arxiv:1012.4708

[22] C. Kiefer, *Quantum Gravity*, 3rd edn. (Oxford UP, 2012)

[23] H. D. Zeh, Emergence of classical time from a universal wave function, Phys. Lett. **A116**, 9 (1986)

[24] C. Kiefer and D. Polarski, Why cosmological fluctuations look classical to us, Adv.Sci.Lett. **2**, 164 (2009) – arxiv:0810.0078 (2008)

[25] M. Tegmark, The Multiverse Hierarchy, arxiv:0905.1283 (2009)

[26] M. Tegmark, How unitary cosmology generalizes thermodynamics and solves the inflationary entropy problem, Phys. Rev. **D 85**, 123517 (2012)

[27] J. C. Baez, Struggles with the Continuum, arxiv:1609.01421 (2016)

[28] A. Chenu, A. M. Branczyk, and J. E. Sipe, Thermal States and Wave Packets, arxiv:1609.00014 (2016)

[29] W. H. Zurek, Quantum Darwinism, Nature Physics, **5**, 181 (2009); C. J. Riedel, W. H. Zurek and M. Zwolak, Objective past of a quantum universe: Redundant records of consistent histories, Phys. Rev. **A93**, 032126 (2016)

– (Refs. 28 and 29 appear in Footnote ** on page 26)