Open questions regarding the arrow of time*

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Abstract: Conceptual problems regarding the arrow of time in classical physics, quantum physics, cosmology, and quantum gravity are discussed. Particular attention is paid to the retardation of various kinds of correlations, the dynamical rôle of the quantum indeterminism, and to different concepts of timelessness.

1. Laws and facts

The Second Law of Thermodynamics is usually regarded as the major physical manifestation of the arrow of time, from which many other consequences can be derived. I have discussed the relations between these different forms of the arrow in detail elsewhere,[1] so I will occasionally refer to this source in the following by TD ("Time Direction") for short. This article is meant to review some open conceptual problems, which are often insufficiently realized, or of actual interest for other reasons.

In Statistical Thermodynamics, the Second Law is derived from the assumption that a closed system must evolve towards a more probable state. In this context, entropy is defined as a measure of probability. This explanation is incomplete for various reasons. First, the concept of evolution already presumes a direction in time. To regard it as a direction *of* time would even apply this asymmetry to the very definition of time. This would go beyond a purely mechanistic concept of time, which is defined in accordance with time-symmetric laws of motion. Newton's absolute "flow of time" is a similar metaphor; its direction would be physically meaningful only if one assumed asymmetric laws. For example, Newton regarded friction as representing a fundamental force that would slow down all motion. So in his opinion God had to intervene once in a while to set things in motion again. Without such an external, metaphysical, or at least law-like fundamental direction in or of time, one can only speak of an asymmetry of the facts (which may well be compatible with symmetric laws).

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Second, the concept of probabilities requires a measure that is usually defined with respect to ensembles of *possible* states. Since in classical physics every system is assumed at any time to be in one definite microscopic state, the latter must be "coarse-grained" in order to define a macroscopic or "thermodynamic state", that is, an ensemble of microscopic states which may thus define a non-trivial probability measure. Various kinds of coarse-graining (omissions of actual or possible information) have been discussed in the rich literature on this subject, or were invented in the context of a new theory. The justification of such ensembles by a macroscopic (that is, incompletely defined) preparation procedure would refer to the time-directed concept of preparations as a *deus ex machina* (similar to Newton's divine interventions). Arguments based on incomplete observability or controllability of certain degrees of freedom may also presume external time-asymmetric observers.

The mechanistic concept of time is usually postulated together with the deterministic dynamical laws that are assumed on empirical grounds to control the facts. Eugene P. Wigner called the distinction between laws and initial conditions (initial facts) Newton's greatest discovery. In a deterministic theory, initial conditions could as well be replaced by final ones, or by conditions at any intermediate time. This mechanistic concept requires only that time can be represented by the real numbers (without any preference for their sign) – thus defining a linear order of physical states or global "Nows". Deterministically, the size of an ensemble (the number of microscopic states, or an appropriate measure if this number is infinite) does not change in time, while its coarse-grained size would, in general. This is why Ludwig Boltzmann's statistical measure H, which up to a factor may be assumed to represent "negentropy" for a diluted gas, may change in time (see Sect. 2). It will decrease even under deterministic equations of motion *in the direction of calculation* – provided the fixed concept of coarse graining was used to define the input ensemble. Further conditions studied in ergodic theory are necessary to exclude exceptional cases that are usually of measure zero. There is no *a priori* reason to calculate only in the conventional "forward" direction of time, but this is *empirically* the only direction in which statistical arguments lead to correct results – thus indicating a strong asymmetry of the facts. For applications to cosmology let me emphasize that the conservation of exact (not coarse-grained) ensemble entropy under deterministic equations of motion would include the situation of a deterministically inflating universe, which has often erroneously be claimed to *cause* a low entropy condition.

Although irreversible phenomena are mostly observed locally, the thermodynamical arrow of time seems to possess a common global direction. Its origin has, therefore, usually been discussed in a cosmological context. For example, one may assume a special cosmic initial condition at the big bang. Boltzmann, who assumed the universe to be eternal, argued instead that a giant chance fluctuation must have occurred in the distant past in order to form a low entropy state. A physical "future" would then be characterized by any time direction away from such a low entropy state. Boltzmann's proposal seems to imply, though, that it would be far more probable to assume that the *present* state of the universe – including all memories and conscious brains – had just formed in a chance fluctuation, since this state would possess very low, but nonetheless much higher entropy than the otherwise required state in the distant past (see, however, Sect. 2). This idea has recently been discussed under the name "Boltzmann brains", mainly in some as yet speculative cosmologies. If, on the other hand, the low entropy is related to a global special condition at high densities, the thermodynamical time arrow would have to change direction in an oscillating universe, while opposite arrows in causally connected parts of the universe seem to be excluded for dynamical reasons.[2]

The arguments based on deterministic dynamics do not directly apply to stochastic dynamics. However, a stochastic law by itself does not necessarily characterize a direction in time. If all states at some time t_1 had two possible successors at a later time t_2, say, and if this law held on all states, then each successor must on average also have two dynamically possible predecessors at time t_1. Therefore, such a stochastic law defines a time-asymmetric indeterminism only when applied to a genuine subset of possible *initial* states, while not restricting the set of final states. This would be just another way of applying the "double standard" that has been duly criticized by Hugh Price.[3] The asymmetry is not a consequence of the stochastic law itself (see Sect. 3.4 of TD and the concept of "forks of indeterminism" mentioned therein). On the other hand, even deterministic laws may be asymmetric, but this would not by itself offer a way to explain the increase of entropy. Examples are the Lorentz force of an external magnetic field or *CP*-violation. In these and similar cases, formal time-reversal symmetry violation is compensated for by another symmetry violation, which may be either physical, such as a *CP* transformation, or just formal, such as complex conjugation in the Schrödinger equation.

Our world is known to obey quantum theory, which is characterized by an indeterminism occurring in measurements and other "quantum events". There is absolutely no consensus among physicists about the interpretation and even the precise dynamical rôle of this "irreversible coming into being" of the observed facts, such as the click of a counter. Has it to be regarded as a specific part of the dynamical laws (as assumed in the form of von Neumann's "first intervention" or more explicitly in collapse theories), as representing events that (according to Wolfgang Pauli) occur outside the laws of nature, as a "normal" increase of information (as claimed in the Copenhagen interpretation), as determined by hidden variables that are not counted in conventional ensemble entropy (as in David Bohm's theory), or as the consequence of inderministically splitting quantum observers (as in Hugh Everett's interpretation)? Some quantum cosmologists refer to initial uncertainty relations or "quantum fluctuations" in order to justify the stochastic evolution of their quantum universe, although a global quantum state is never required to be "uncertain" (only classical variables had to be assumed to be uncertain if they were used).

In the pragmatic Copenhagen interpretation, this problem is essentially circumvented by denying any microscopic reality, while other above-mentioned proposals suggest novel laws or concepts, which may or may not be confirmed or ruled out in principle. Although these various interpretations must have drastic consequences for the resulting model of the universe, they play surprisingly almost no rôle in actual cosmology. For example, the thermodynamical arrow might be the consequence of a time-asymmetric collapse mechanism if this were part of the laws. In the Copenhagen interpretation, there simply "is no quantum world" – hence no complete cosmological model. Most cosmologies are therefore based on classical concepts, just allowing for some "quantum corrections", while indeterministic master equations are often derived from unitary equations of motion by using certain "approximations" in analogy to classical statistical physics. Such equations may then even *appear to explain* stochastic and irreversible quantum events, although they are implicitly using them.

Much philosophical debate has also been invested into the pseudo-distinction between a block universe and an evolving universe (a world of being versus a world of becoming). However, these apparently different pictures describe only different representations of the same thing. One should realize that a block universe picture is by no means restricted to a physical context. Historians have always been applying it to the past, although they never had doubts that Cesar crossed the Rubicon according to his free will. We can similarly use space-time diagrams to represent *actual* motions or *potential* histories (individual members of an ensemble of possible histories) even in the case of an indeterministic law. Moreover, a block universe picture has nothing specifically to do with relativity (except that it is just convenient in the absence of a concept of absolute simultaneity).

2. The arrow in classical physics

It is essential to keep in mind that time-symmetric laws are perfectly compatible with asymmetric solutions. Almost all solutions of the fundamental equations of motion are time-asymmetric, while reasonably defined quasi-recurrence times for isolated systems would exceed the age of the universe by enormous factors. The symmetry of the laws of motion requires only that for every asymmetric solution that is realized in nature there must mathematically – not necessarily physically – exist precisely another, time-reversed one. In reality, though, very few systems can be considered as being isolated.[4] This means that the reversed solution would require an exact time reversal of its complete environment – an argument that must then be extended to the whole causally connected region of our universe. An extremely small “perturbation” (change of the state at some time) would with overwhelming probability turn a deterministic solution with decreasing entropy into one with increasing entropy (in both directions of time).[2]

Remarkable is only that there are whole classes of asymmetric solutions that are found in abundance, while members of the reversed class are rarely or never observed. As an example, consider the contrast between retarded and advanced Maxwell fields for a given type of source. This asymmetry may be understood as a consequence of the presence of absorbers (including the early radiation era of our universe). Absorbers are based on the thermodynamical arrow of time, since they describe the transition to thermal equilibrium between radiation and matter. So they produce “retarded shadows”, which, when forming a complete spatial boundary, give rise to local initial conditions of no incoming radiation at frequencies above the thermal spectrum (see Chap. 2 of TD). But why do all physical absorbers absorb in one and the same direction of time only?

The precise microscopic states of systems consisting of many interacting constituents can hardly ever be *known* even in a classical world. So it is common practice to use an incomplete description for them (a generalized coarse-graining). For example, a gas may be described by the mean phase space distribution $\rho_\mu(\boldsymbol{p},\boldsymbol{q},t)$ of its molecules. Its evolution in the forward direction of time is then successfully described by Boltzmann’s stochastic collision equation. This asymmetric success must be a consequence of properties of the thereby neglected *correlations* between molecules, since the increase of Boltzmann’s entropy S_B,

(1) $$S_B := -Nk_B\overline{\ln\rho_\mu} = -Nk_B\int\rho_\mu\ln\rho_\mu d^3p d^3q \quad ,$$

where k_B is Boltzmann's constant and N the particle number, can be deterministically understood as a dynamical transformation of information represented by the μ–space distribution into information about correlations. Both kinds of information are described by the 6N-dimensional Γ-space distribution ρ_Γ, whereby the analogously defined ensemble entropy S_Γ does *not* change under deterministic dynamics. While dynamical models readily confirm that correlations produced in a scattering process remain irrelevant for ρ_μ for an extremely long time, one has to assume asymmetrically that only "retarded correlations", required to reproduce the past, are relevant for the single-particle distribution. This absence of advanced correlations is even "probable", while the low-entropy initial condition that leads to retarded correlations – such as a special initial μ-space distribution ρ_μ – is *not*. Explaining this asymmetry by referring to "causality" would beg the question.

There are many appropriate ways to distinguish between macroscopic and microscopic ("irrelevant") degrees of freedom. They can all be formally described by some idempotent "Zwanzig" operator P that acts on the Γ-space distributions $\rho = \rho_\Gamma$ (see Sect. 3.2 of TD),

(2) $$\rho = P_{rel}\,\rho + P_{irrel}\,\rho\ , \qquad \text{with} \quad P_{rel}^{\ 2} = P_{rel} \quad \text{and} \quad P_{irrel} = 1 - P_{rel}\ ,$$

where the macroscopically relevant part, $\rho_{rel} = P_{rel}\rho$, defines a generalized "coarse-grained" distribution. Macroscopic properties are characterized by a certain robustness or controllability, which may vary with the physical situation. For example, correlations between molecules or ions are stable and relevant in solid bodies, while the corresponding lattice vibrations can then mostly be treated thermally. Although the exact dynamics requires a coupling between ρ_{rel} and ρ_{irrel}, there often exists a probabilistic effective "master equation" for ρ_{rel} that reflects the dynamical future irrelevance of ρ_{irrel} for the dynamics of ρ_{rel}, as exemplified by Boltzmann's collision equation, where ρ_{rel} can be defined in terms of ρ_μ.

The physically appropriate relevance concept used to define ρ_{rel} may thus change in time. In such cases, the usual ignorance of microscopic degrees of freedom can be deterministically transformed into "lacking information" about arising macroscopic ("relevant") ones – such as the positions of droplets formed during a condensation process. This happens, in particular, in symmetry-breaking phase transitions, or in measurements of microscopic variables (but these processes assume a completely new form in quantum theory). Strictly speaking, only the complete ensemble entropy, measured by the mean logarithm of ρ_Γ itself (without any coarse-graining), is conserved under deterministic equations of motion. Physical entropy is usually defined *not* to include that part which represents lacking information about macroscopic variables, but rather as a *function* of them. However, the transformation of physical entropy into entropy of lacking information about variables that are usually assumed to be "physically given" cannot be used in a cyclic process to construct a perpetuum mobile of the second kind.[5] Although the formal entropy of lacking information is in general thermodynamically negligible, it may become essential for fundamental considerations – such as those involving Maxwell's demon. While the precise definition of entropy (its specific relevance concept or Zwanzig projection) is in principle a matter of convenience, the initial cosmic low-entropy condition that would "cause" an arrow of time must represent a specific property of the universe, and its precise nature should therefore be revealed.

The robustness of macroscopic properties together with the retardation of all correlations between them means that there are many redundant macroscopic "documents" (including fossils and personal memories) about the macroscopic past. The latter is therefore said to be "overdetermined" by the macroscopic present or future.[6] It appears fixed because it could not have been different if just one (or a few) documents were found to be different. Precisely this *consistency of the documents* makes them trustworthy and distinguishes them from mere chance fluctuations with the same low value of physical entropy. Julian Barbour has called states that contain consistent documents (regardless of their causal origin) "time capsules".[7] Since conventional concepts of physical entropy are local (based on an entropy *density*), they cannot distinguish between consistent and inconsistent documents. An evolved ("historical") state has much lower statistical probability than indicated by its physical entropy, and this fact may rule out Boltzmann brains for being "statistically unreasonable" (see Sect. 3.5 of TD).

In most cosmological models, the low-entropy initial condition is represented by a "simple" state of high symmetry – very different from a later state of still low but larger entropy that describes complexity and dynamical order as it exists in organisms, for example. While an exactly symmetric state could not evolve into an asymmetric one by means of symmetric and deterministic laws, a state consisting of classical particles cannot be *exactly* (microscopically) homogenous: the information capacity of a single continuous variable is infinite, and any exact value of a spatial variable would violate this symmetry. Nonetheless, a Laplacean universe that is symmetric after appropriate coarse-graining may determine all later arising complexity.

While, in a laboratory situation, thermal equilibrium normally requires macroscopically homogeneous ensembles, such states are still extremely improbable (and hence unstable) in self-gravitating systems. Gravitating stars and galaxies, for example, possess negative heat capacity: they become hotter and denser when losing energy (see Sect. 5.1 of TD). Classically, this negative heat capacity would even be unbounded. Therefore, the initial homogeneity of the universe is a major candidate for the specific low entropy condition that characterizes this universe. Roger Penrose has formulated this condition in general relativity by postulating a vanishing Weyl tensor on all past singularities. This source-free part of the spacetime curvature tensor can be interpreted as representing gravitational radiation. The Weyl condition would thus mean that all gravitational radiation must be retarded (possess sources in its cosmic past that begins at the singularity). An analogous condition had been proposed for electromagnetic radiation by Planck in a debate with Boltzmann, and later by Ritz in a debate with Einstein. However, because of the weak coupling of gravity to matter, the Weyl tensor condition cannot similarly be explained by the thermodynamic properties of absorbing matter. It may then itself establish the causal nature of the universe, that is, be responsible for the absence of future-relevant early correlations.

3. The arrow in quantum theory and quantum cosmology

Although the quantum formalism of irreversible processes is formally quite analogous to its classical counterpart (see Sect. 4.1 of TD), there are at least three genuine quantum aspects that are important for the arrow of time: (1) the superposition principle, (2) a quantum indeterminism of controversial origin – often described by a collapse of the wave function, and (3) quantum nonlocality – a specific consequence of (1).

The superposition principle allows *exactly* symmetric elementary states for all kinds of symmetries. Such symmetric states may then form candidates for an entirely unspecific initial pure state. Although they cannot unitarily evolve into asymmetric states by means of a symmetric Hamiltonian, they could do so by means of an appropriate indeterministic collapse of the wave function that does not obey the principle of sufficient reason. While such a collapse has always to be used *in practice* in order to describe measurements or phase transitions in terms of quantum states, a non-unitary modification of the Schrödinger equation that would satisfactorily describe it in a general way has never been experimentally confirmed. Therefore, Everett's "branching" of the quantum universe (including all observers) into different autonomous components describing quasi-classical "worlds" must be taken seriously as forming an alternative – whatever it means. This branching is objectively specified by an in practice irreversible decoherence process that is described by the Schrödinger equation.

While the collapse would define a time-asymmetric law, the time arrow of decoherence (formation of retarded entanglement) must again arise as a consequence of an initial condition – now for the global wave function. A universally valid Schrödinger equation would in principle also admit the anti-causal process of recoherence, but this is very rare under an appropriate initial condition. Although any initial symmetry of the global state must be conserved under a symmetric Hamiltonian, a non-entangled ("simple") symmetric state can evolve into a symmetric superposition of many asymmetric Everett branches (independent "worlds" possessing a complex structure). This subtlety is neglected in many quantum cosmological models – in particular when other formally arising Everett branches are simply disregarded for being "meaningless". Unitarily calculating backwards in time, however, would require knowledge of *all* Everett branches or collapse components (including the unobserved ones) *and their phase relations* as an input. While the macroscopic past ("history") is overdetermined by the present even in an individual branch, the microscopic past is underdetermined even if all present branches (in conventional laguage "possibilities" that *could* have occurred) were known independently of one another.

A stochastic collapse by itself (that is, when neglecting the accompanying decoherence processes) would *reduce* nonlocal entanglement, since it is usually defined to select components that factorize in the relevant subsystems (see Sects. 4.6 and 6.1 of TD). This consequence applies as well to the transition into an *individual* Everett world that is experienced by local (themselves branching) observers which are in definite states. Such a dynamical reduction of entanglement is required, in particular, in order to obtain definite outcomes in measurement-like processes, or to allow the preparation of pure initial states in the laboratory or during a process of self-organization.

This indeterministic transition into less entangled states must reduce any physical entropy measure that is defined by means of a Zwanzig projection of locality (as required if entropy is to be an extensive quantity). It is here important to recognize the difference between *classical* microscopic states, which are local by definition (that is, they are defined by the states of all their local subsystems), and generically nonlocal quantum states. Therefore, the physical (local) entropy of a completely defined ("real") classical state is minimal (minus infinity), while that of a pure quantum state is not only non-negative, but in general also much greater than zero (non-trivial). The permanent creation of uncontrollable quantum entanglement by decoherence must dominate the creation of physical entropy, which would for large times lead to those apparent local ensembles (improper mixtures) that represent thermodynamical equilibrium. It is tacitly used in phenomenological "open systems quantum mechanics". The *reduction* of entropy in a process of symmetry breaking, on the other hand, is usually very small when compared with thermodynamic entropy, but it may be cosmologically essential when, for example, it leads to new Goldstone type particles that usually possess an enormous entropy capacity (see Sect. 6.1 of TD).

Another novel consequence of quantum theory that regards the arrow of time is the entropy bound that governs gravitational contraction. It is characterized by the Bekenstein-Hawking black hole entropy, given by

$$S_{BH} = 4\pi \frac{k_B G M^2}{\hbar c} \tag{3}$$

for spherical and electrically neutral black holes. Here, G is the gravitational constant. The fact that S_{BH} is quadratic in the mass M indicates that it must describe some kind of correlations. According to classical general relativity, spacetime geometry is regular at the black hole horizon, while there has to be a future singularity inside. However, the interior cannot causally affect the external region any more: it must for all times remain in the future of all external observers. This leaves much freedom for the unknowable physics inside. In particular, *quantum* gravity does not allow one to distinguish between past and future singularities any more (see below). Therefore, one can only postulate a Weyl tensor condition on *all* space-like singularities. Such a time-symmetric condition is not only compatible with all observations – it may even prevent black hole interiors and horizons to form (thus avoiding any genuine information loss paradox).[8]

Most of these genuine quantum aspects of the cosmic arrow of time have so far received little attention – perhaps because they seem to depend on the interpretation of the quantum formalism. Cosmological models are mostly presented in classical terms rather than in terms of quantum states (superpositions). In particular, arguments based on Feynman's path integral often replace this integral, which describes a superposition of paths (precisely equivalent to a wave function[9]) by an *ensemble* of paths in classical configuration space. Selecting a sub-ensemble or an individual path from them is nonetheless equivalent to a time-asymmetric collapse of the wave function. Similar objections apply to tunneling probabilities, since any decay process must quantum mechanically be described as a coherent superposition of different decay times as long as the corresponding partial waves are not irreversibly decohered from one another (thereby letting decay events appear to be "real" rather than virtual – Sect. 4.5 of TD).

A consistent quantum description requires that classical general relativity is replaced by quantum gravity. This does not necessarily require a complete understanding of this theory. While the problem of the arrow of time can probably be finally answered only in an ultimate theory, the meaning and validity of existing proposals (such as in string theories) have remained highly speculative as yet. Standard quantization of the canonical form[10] of General Relativity in the Schrödinger picture, on the other hand, leads to the Wheeler-DeWitt equation (or Hamiltonian quantum constraint),[11]

$$H\,\Psi = 0\ , \qquad (4)$$

which may be expected to form an effective theory of quantum gravity at "low" (that is, normal) energies. The wave functional Ψ depends on spatial geometries and matter fields on arbitrary simultaneities. Since the Schrödinger equation now takes the form $\partial\Psi/\partial t = 0$, there exists no time parameter any more that could be used to formulate a direction in time. This "timelessness" has occasionally been regarded as a severe blow to this approach, although it must apply to all quantum theories that are reparametrization invariant in their classical form. However, the physical concept of time – and even its arrow – can be recovered and understood in a satisfactory way under very reasonable assumptions.[12]

The first important observation for this purpose is that the Wheeler-DeWitt equation for Friedmann type universes is globally of hyperbolic type, with a time-like variable $\alpha := \ln a$, where a is the cosmic expansion parameter.[13] This fact defines an intrinsic "initial" value problem in α or a, for example at the big bang ($\alpha = -\infty$), which would in configurations space be identical with a big crunch. The Wheeler-DeWitt equation is drastically asymmetric under a change of sign of α, thus suggesting an asymmetric solution without explicitly postulating it by means of asymmetric boundary conditions. The second step for recovering conventional time is a Born-Oppenheimer expansion in terms of the inverse Planck mass, which equals 1.3 10^{19} proton masses.[14] This mass characterizes all geometric degrees of freedom. The expansion leads to an approximately autonomous evolution of partial Wheeler-DeWitt wave functions for the matter degrees of freedom along WKB trajectories that are defined in most regions of the configuration space of geometries. This is analogous to the adiabatic evolution of electron wave functions along classical orbits of the heavy nuclei in large molecules. This evolution has precisely the form of a time-dependent Schrödinger equation (plus very small corrections).[15] The concept of time recovered in this way represents arbitrary time coordinates for all possible foliations, and independently for all dynamically arising quasi-classical spacetimes (branches).

Note that this WKB approximation does not by itself justify an *ensemble* of trajectories, since it preserves the global superposition that they form. Similarly, small molecules (for which the positions of nuclei are *not* decohered to become quasi-classical variables) are known to exist in energy eigenstates (wave functions) in spite of the validity of the Born-Oppenheimer approximation. However, since observers would also possess different states in the different autonomous partial waves for the universe, they can observe only their own "branch" as an apparently evolving quantum world. The global intrinsic dynamics would be required, though, in order to dynamically *derive* the initial conditions for all partial Schrödinger wave functions that have to be used in the WKB region of geometry (at some distance from the big bang).

According to arguments used in loop quantum cosmology, the Wheeler-DeWitt equation (in this theory replaced by a difference equation with respect to a) can be continued through $a = 0$ to negative values of a.[16] The configuration space of three-geometries is in this way duplicated by letting the volume measure assume negative values (turning space "inside out" while going through $a = 0$). Since the Hamiltonian does not depend on the newly invented sign of a, however, the Wheeler-DeWitt wave function must be expected to be symmetric under this parity transformation, too, in the absence of any artificial boundary condition. Its continuation would then have to be interpreted as an enlarged superposition of components that are all individually experienced as *expanding* universes. Since their WKB times, which represent classical times, can *not* be continued through $a = 0$, where the WKB approximation breaks down, the interpretation of negative values of a as representing pre-big-bang times is highly questionable. The fundamental arrow, including its consequence of decoherence with respect to a even outside the validity of a WKB approximation, must depend on some low entropy (no entanglement) "initial" condition in this time-like variable for all other ("spacelike") degrees of freedom that occur as physical arguments of the Wheeler-DeWitt wave function. It would be hard to understand how the low entropy state at $a = 0$ could have been "preceded" by an even lower entropy at $a < 0$ in order to avoid a reversal of the thermodynamical arrow in the classical picture of an oscillating universe.

In spite of the success in recovering physical time for the autonomous Everett branches that represent quasi-classical spacetimes, "timelessness" has recently become a hot issue that is based on some severe misunderstandings. It has even been used as a motivation to present obscure and speculative solutions to this non-existing problem. I will, therefore, now give a brief review of *different* concepts of timelessness that have been discussed and confused in this connection.

4. A brief history of timelessness

Newton described planetary motions in the form $r(t)$, $\varphi(t)$ that required a concept of absolute time. He also assumed, by means of his laws, that absolute time t can be read from appropriate clocks, such as the rotation of the Earth, $\alpha = \omega t$. Elimination of t from the first two functions leads to Kepler's orbits $r(\varphi)$. Similarly, its elimination from all three functions leads to a clock dependence $r(\alpha)$ and $\varphi(\alpha)$. This trivial elimination of time has recently been used by some authors to argue that one should "forget time" in all dynamical considerations.[17] However, this argument completely neglects the fact that it is precisely Newton's time that simplifies his laws of motion, as has been clearly emphasized by Henri Poincaré. So, in Newtonian physics there is a preferred time parameter that could indeed be interpreted as representing "absolute" time.

The concept of absolute time was not only questioned for philosophical reasons by Leibniz and Mach, it also lost its empirical justification in General Relativity. In Special Relativity, absolute time is replaced by an absolute spacetime metric that still defines path-dependent proper times. According to the principle of relativity, they control all physical motions in the same preferred way as Newtonian time did in non-relativistic physics. In particular, local clocks *measure* proper times along their world lines, while the spacetime metric is assumed to exist even in the absence of physical clocks.

In General Relativity, the spatial metric defined on arbitrary simultaneities becomes itself a dynamical object[10] – just as any matter field. Its evolution gives rise to a succession of spatial curvatures that defines a foliation of spacetime. It can be parametrized by an arbitrarily chosen time coordinate, but there is no *preferred* coordinate or time parameter any more. Julian Barbour has discussed this Machian property, which he called timelessness, in great detail, including many consequences that were historically important.[18] However, the arising metric still defines proper times for all world lines (Wheeler's *many-fingered time*), and the evolving spatial metric can itself be regarded as a many-fingered *physical* clock.[19] Although there are many different time-like foliations of the same spacetime, each one defines a parametrizable succession of states, and this dynamical construction allows in general the formulation of a unique initial value problem – hence an initial condition of low entropy. There are also mathematically consistent non-relativistic Machian ("relational") theories.[20]

The complete absence of any time parameter from the Wheeler-DeWitt wave function (genuine timelessness), discussed in the previous section, is a specific *quantum* property: it is a consequence of the fact that in quantum theory there are no trajectories (in configuration space) that could be parametrized. Hence, in quantum gravity there are no classical space-times that could give rise to a time-like foliation. There is only a probability amplitude for spatial geometries (many-fingered physical clocks) entangled with matter fields.[21] For the same reason, the recently proposed concept of "relational observables"[17] is inappropriate, since it is based on a classical concept of orbits, required to define such relations between variables. Entanglement describes also the decoherence of macroscopically different geometries from one another if matter is regarded as an environment to geometry.[12] Among the parameters characterizing these spatial geometries is the "intrinsic time" $\alpha = \ln a$. It is remarkable that this genuine timelessness (the inapplicability of *any* external time parameter) was known before its weaker classical versions were discussed under this ambitious name. Unfortunately, it seems to have initially been mostly regarded as a merely formal problem. The reason may be that early physicists working on quantum gravity did not take the Wheeler-DeWitt wave function seriously as representing reality. They either used semi-classical approximations for its interpretation (even where they were not justified), or they preferred a Heisenberg picture, in which the problem is less obvious.[22]

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