The strange (hi)story of particles and waves*

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Abstract: This is an attempt of a non-technical but *conceptually consistent* presentation of quantum theory in a historical context. While the first part is written for a general readership, Sect. 5 may appear a bit provocative. I argue that the single-particle wave functions of quantum mechanics have to be correctly interpreted as *field modes* that are "occupied once" (that is, first excited states of the corresponding quantum oscillators in the case of boson fields). Multiple excitations lead non-relativistically to apparent many-particle wave functions, while the quantum states proper are always defined by wave function(al)s on the configuration space of fundamental fields, or on another, as yet elusive, fundamental local basis.

Sects. 1 and 2 are meant as a brief overview of the early history - neglecting details. Sects. 3 and 4 concentrate on some important properties of non-relativistic quantum mechanics that are insufficiently pointed out in many textbooks (including quite recent ones). Sect. 5 describes how this formalism would have to be generalized into its relativistic form (QFT), although this program generally fails in practice for *interacting* fields because of the complicated entanglement that would arise between too many degrees of freedom. This may explain why QFT is mostly *used* in a semi-phenomenological manner that is often misunderstood as a fundamentally new theory. Sect. 6 describes the application of the Schrödinger picture to quantum gravity and quantum cosmology, while Sect. 7 concludes the paper.

1. Early History

The conceptual distinction between a discrete or a continuous structure of matter (and perhaps other „substances") goes back at least to the pre-Socratic philosophers. However, their concepts and early ideas were qualitative and speculative. They remained restricted to some general properties, such as symmetries, while the quantitative understanding of continuous matter and motion had to await the conceptual development of calculus on the one hand, and the

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– Depending on your interests, you may prefer to skip the first one or two sections!

availability of appropriate clocks on the other. Quantitative laws of nature and the concept of mass points, for example, were invented as part of classical mechanics.

This theory was first applied to extended “clumps of matter”, such as the heavenly bodies or falling rocks and apples. It was in fact a great surprise for Newton and his contemporaries (about 1680) that such very different objects – or, more precisely, their centers of mass – obeyed the same laws of motion.[1] The objects themselves seemed to consist of continuous matter, although the formal concept of mass points was quite early also applied to the *structure* of matter, that is, in the sense of an atomism. Already in 1738, Daniel Bernoulli explained the pressure of a gas by the mean kinetic energy of small objects, but without recognizing its relation to the phenomenon of heat. If one regarded these objects themselves as small elastic spheres, however, the question for their internal structure would in principle arise anew. The concept of elementary particles thus appears problematic from the outset.

At about the same time, Newton’s theory was also generalized by means of the concept of a continuum of infinitesimal mass points which can move according to their local interaction with (that is, mainly repulsion by) their direct neighbors. This route to continuum mechanics required novel mathematical concepts, but no fundamentally new *physical* ones beyond Newton. The assumption of an unlimited divisibility of matter thus led to a consistent theory. In particular, it allowed for wave-like propagating density oscillations, required to describe the phenomenon of sound. So it seemed that the fundamental question for the conceptual structure of matter had been answered.

As a byproduct of this “substantial” (or “Laplacean”) picture of continuum mechanics, based on the assumption of distinguishable and individually moving infinitesimal elements of matter, also the elegant “local” (or “Eulerian”) picture could be formulated. In the latter, one neglects any reference to trajectories of individual elements in order to consider only its spatial density distribution together with a corresponding current density as the kinematical objects of interest. In modern language they would be called a scalar and a vector *field*. In spite of this new form, continuum mechanics thus remains based on the concept of a locally conserved material substance.

The model of a coherent continuum of mass points would prove incomplete, however, if the latter could move freely, interrupted only by occasional collisions, as suspected for a gas by Daniel Bernoulli. Since his gas pressure is given by the density of molecular kinetic energy, that is, by the product of the number density of gas particles and their mean kinetic energy, this picture could nonetheless be understood as representing a “chaotic continuum” by means of an appropriately defined simultaneous limit of infinite particle number density and vanishing particle mass. This remained a possibility even when chemists began to successfully apply Dalton’s and Avogadro’s hypotheses about molecular structures from the beginning of the nineteenth century in order to understand the chemical properties of the various substances. Similar to Auguste Bravais’s concept of crystal lattices (about 1849), these structures were often regarded as no more than a heuristic tool to describe the internal structure of a multi-component continuum. This view was upheld by many even after Maxwell’s and Boltzmann’s explanation of thermodynamic phenomena in terms of molecular kinetics, and in spite of repeated but until then unsuccessful attempts to determine a finite value for Avogadro’s or Loschmidt’s numbers. The “energeticists”, such as Wilhelm Ostwald, Ernst Mach and initially also Max Planck remained convinced until about 1900 that atoms are an illusion, while concepts like internal energy, heat and entropy would describe fundamental continua. Indeed, even after the determination of Loschmidt’s number could they have used an argument that formed a severe problem for atomists: Gibbs’ paradox of the missing entropy of self-mixing of a gas. Today it is usually countered by referring to the indistinguishability of molecules of the same kind, while the argument requires more, namely the *identity* of states resulting from permutations. Such an identity would be in conflict with the concept of particles with their individual trajectories, while a field with two bumps at points x and y would trivially be the *same* as one with bumps at y and x. Although we are using quite novel theories today, such conceptual subtleties do remain essential (see Sect. 5).

Another object affected by the early dispute about particles and waves is light. According to its potential of being absorbed and emitted, light was traditionally regarded as a “medium” rather than a substance. Nonetheless, and in spite of Huygens’ early ideas of light as a wave phenomenon in analogy to sound, Newton tried to explain it by means of “particles of light”, which were supposed to move along trajectories according to the local refractive index of matter. This proposal was later refuted by various interference experiments, in particular those of Thomas Young in 1802. It remained open, though, what substance (called the ether) did oscillate in space and time – even after light had been demonstrated by Heinrich Hertz in 1886 to represent an electromagnetic phenomenon in accordance with Maxwell’s equations. The possibility of these fields to propagate and carry energy gave them a certain substantial character that seemed to support the world of continua as envisioned by the energeticists. Regarding atoms, Ernst Mach used to ask “Have you ever seen one?” whenever somebody mentioned them to him. Later in this article I will argue that his doubts may be justified even today – although we *seem* to observe individual atoms and particle tracks. Similar to the phenomenon of "events" or "quantum jumps", they may be an illusion caused by the dynamics of the wave function.

At the end of the nineteenth century, the continuum hypothesis suffered a number of decisive blows. In 1897, J. J. Thomson discovered the elementary electric charge; in 1900, Max Planck postulated his radiation quanta for the electromagnetic field with great success; and in 1905, Albert Einstein estimated the value of Loschmidt's number N_L by means of his theory of Brownian motion. Thereafter, even the last energeticists resigned, but they left some confusion about the concept of a physical "state". While they had regarded temperature, pressure or internal energy density etc. as locally characterizing the ontic state of matter, in atomistic description, these "thermodynamic states" require some averaging over the true and fundamental states, either in time, or in space ("coarse graining"), or with respect to some incomplete knowledge. In quantum theory, this confusion about states survives in the program of a pure operationalist approach and in the concept of "mixed states" (see Sect. 4).

Einstein then even revived the concept of particles of light (later called photons) – although he regarded it merely as a "heuristic point of view" that he was never ready to fully accept himself. For a long time, Plancks radiation quanta were indeed attributed to a discrete emission process rather than to the radiation itself. So in 1913, Niels Bohr replaced the concept of classical motion for atomic electrons by stochastic "jumps" between his discrete orbits – in accordance with Planck's and Einstein's ideas about a probabilistic radiation process. These early ideas led later to the insufficient interpretation of quantum mechanics as no more than stochastic dynamics for otherwise classical particles.

However, the development soon began to proceed in the opposite direction again.[2] In 1923, Louis de Broglie inverted Einstein's speculative step from light waves to photons by postulating a wave length $\lambda = c/\nu = h/p$ for the electron, where p is its momentum, in analogy with Planck's relation $E = pc = h\nu$. For him, this could only mean that all microscopic objects must consist of both, a particle *and* a wave, whereby the wave has to serve as a "guiding field" or "pilot wave" for the particle. This field would have to be more powerful than a conventional *force* field, since it has to determine the velocity rather than merely the acceleration; the initial velocity can according to this proposal not be freely chosen any more. When David Bohm later brought this theory into a consistent form, it turned out that the pilot wave cannot be defined in space ("locally"), since it has to be identified with the global entangled wave function to be described in Sect. 4.

2. Wave Mechanics

Inspired by de Broglie's ideas, Schrödinger based his novel wave mechanics of 1926 on the assumption that electrons are *solely* and uniquely described by wave functions (spatial fields, as he first thought). His wave equation allowed him to explain the hydrogen spectrum by replacing Bohr's specific electron orbits in the atom by standing waves. This allowed him to explain the puzzling discrete quantum numbers by the numbers of nodes the wave function needs to obey its boundary conditions. For a special case, the harmonic oscillator, he was furthermore able to construct "wave packets" that may imitate *moving* particles (see Fig. 1 for the case of *free* motion, however). Shortly thereafter, interference phenomena in agreement with de Broglie's wave length were observed by Davisson and Germer for electrons scattered from crystal lattices. A wave function can furthermore penetrate a potential barrier and thus explain "quantum tunneling", required for the possibility of α-decay. Does this not very strongly indicate that electrons and other "particles" are in reality just wave packets of some fields that obey Schrödinger's wave equation?

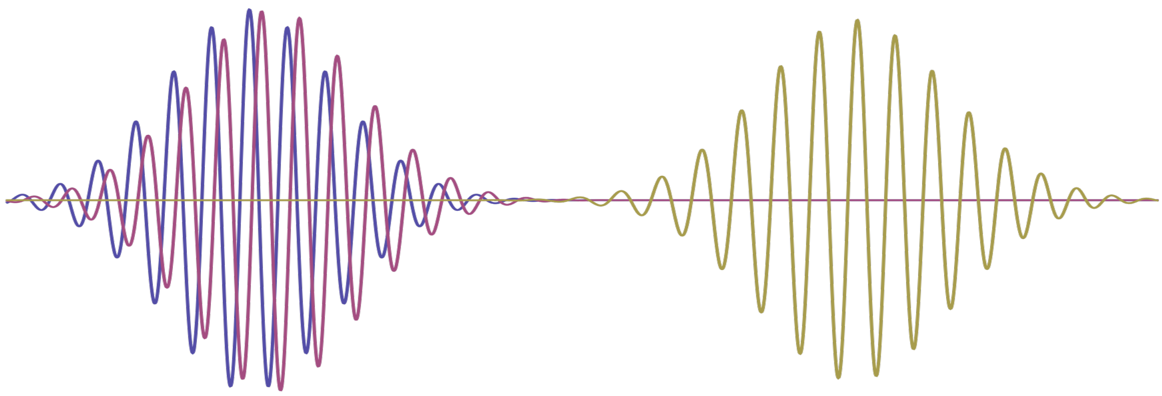

Fig. 1: Real part of a one-dimensional complex wave packet (the product of a Gaussian with a plane wave $e^{2\pi ix/\lambda}$) moving freely according to the time-dependent Schrödinger equation, depicted at three different times (blue: t=0, red: t=0.04, yellow: t=1 in relative units). If the wave packet describes reality, its width defines a "real uncertainty" for the object position; it does neither represent incomplete information, nor is it related to the measureable "particle" size (which has to be described by *internal* degrees of freedom – see Sect. 4). When comparing blue and red, one recognizes that the packet moves faster than its wave crests, while the yellow curve demonstrates a slight dispersion of the packet (in contrast to that of the mentioned harmonic oscillator). The center of the packet moves according to the group velocity $v = p/m := h/m\lambda$, where the mass m is just a parameter of the wave equation. For this reason, momentum is in wave mechanics *defined* by the wave number h/λ (not by motion!), although it is mostly *observed* by means of such moving wave packets (moving "objects"). It can be measured even for plane waves, which would not define a group velocity, by means of the conservation of wave numbers $k = 2\pi/\lambda$ for interactions with objects that do exist as wave packets, thus giving rise to the concept of "momentum transfer". Already for atomic masses and thermal velocities, the de Broglie wave length is clearly smaller than

the radius of a hydrogen atom, so one may construct quite narrow wave packets for their center of mass (cms) wave functions. While the dispersion of the wave packet decreases with increasing mass m, it becomes always non-negligible after a sufficient time interval. In order to compensate for it, one would need an additional dynamical mechanism that permanently reduces the "coherence length" characterizing a wave packet in order to retain the appearance of a particle (see for "collapse" or "decoherence" in Sect. 4).

A few months before Schrödinger invented his wave mechanics, Heisenberg had already proposed his matrix mechanics. In contrast to Schrödinger, he did not abandon the concept of particles, but in a romantic attempt to revive Platonic idealism and overcome a mechanistic world view, combined with an ingenious guess, he introduced an abstract formalism that was to replace the concept of deterministic trajectories by formal probabilistic rules. Together with Born and Jordan, Heisenberg then constructed an elegant algebraic framework that could be used to "quantize" all mechanical systems. This mathematical abstraction perfectly matched Heisenberg's idealistic philosophy. In particular, matrix mechanics was shown to lead to the same discrete "eigenvalues" (distinguished by "quantum numbers") as wave mechanics – although in a less intuitive way. A year after his first paper, Heisenberg supplemented his formalism by his uncertainty relations between position and momentum of an electron or other "conjugate" pairs of variables. Such a fundamental uncertainty is clearly in conflict with a consistent concept of particles, while in wave mechanics it would simply be a consequence of the Fourier theorem – without any uncertainty *of the wave function* or the assumption of an unavoidable "distortion" of the state of the electron during a measurement (as originally suggested by Heisenberg). Another indication of a choice of inappropriate concepts may be the requirement of a "new logic" for them. So it is not surprising that Schrödinger's intuitive wave mechanics was preferred by most atomic physicists – for a short time even by Heisenberg's mentor Max Born. For example, Arnold Sommerfeld wrote only a "Wellenmechanischer Ergänzungsband" to his influential book "Atombau und Spektrallinien".

Some important phenomena, though, remained in conflict with Schrödinger's theory. While his general wave equation $i\hbar\partial\psi/\partial t = H\psi$ would allow various time-dependent solutions, such as the moving wave packet of Fig. 1, bound electrons appeared to be restricted to standing waves. The latter are solutions of the stationary Schrödinger equation $H\psi = E\psi$ that gives rise to the observed discrete eigenvalues E_n under the required boundary conditions. Although this equation can be derived from the general one under the assumption of a special time dependence of the form $\psi \propto e^{iEt/\hbar}$, there is no obvious reason for this special form. Instead of obeying the time-dependent equations, these bound states seemed to be dynamically related by Bohr's stochastic "quantum jumps", which would thus explain energy quanta of radiation (including the hydrogen spectrum) by means of the conservation of energy. Other wave functions seem to "jump" or "collapse" into particle-like narrow wave packets during position measurements. In a Wilson chamber, one could even observe tracks of droplets that can be regarded as successions of such position measurements along particle trajectories.

As a consequence, Schrödinger seemed to resign when Max Born, influenced by Wolfgang Pauli, re-interpreted his new probability postulate, which originally was to describe jumps between different wave functions, in terms of probabilities for the spontaneous *creation of particle properties* (such as positions or momenta). This interpretation turned out to be very successful (and earned Born a Nobel prize) even though it was never quite honest, since the wave function does *not only* describe probabilities. It is also required to represent individual observable properties, such as energy or angular momentum, by means of corresponding "eigenstates", whose spatial structure can often be confirmed by experiments. Similarly, a spinor (a generalized wave function for the electron spin) describes probabilities for other *individual* spinor states rather than for classical properties.

The impossibility to derive the successful wave function from his uncertainty principle (while the reverse *is* possible) was so painful for Heisenberg that he regarded the former as "a new form of human knowledge as an intermediary level of reality", while Bohr introduced his, in his own words, "irrational" principle of complementarity. It required the application of mutually exclusive ("complementary") classical concepts, such as particles and waves, to the same objects. No doubt – this was an ingenious pragmatic strategy to avoid many problems, but, from there on, the search for a consistent description of Nature was not allowed any more in microscopic physics. Pure *Gedanken*-experiments, traditionally used as consistency tests for physical concepts, were now discredited for being "counterfactual". As an answer to the question whether the electron be *really* a wave or a particle (or what else), Bohr insisted that "there is no microscopic reality" – a conclusion that was often regarded as philosophically very deep. Only few dared to object that "this emperor is naked", and the term "complementarity" no more than a new name for a conceptual inconsistency. The large number of attempts of a philosophical or formal explanation of this "nonconcept" is even the more impressive. Furthermore, the question when and where precisely the probability interpretation (or the "Heisenberg cut" between quantum and classical concepts) has to be applied, remained open to be pragmatically decided from case to case. Therefore, the Hungarian Eugene Wigner spoke of a "Balkanization of physics" – a traditional (Hapsburgian) expression for the deterioration of law and order at those times.

3. Wave Functions in Configuration Space

So one should take a more complete look at Schrödinger's wave mechanics. When he formulated it, he used Hamilton's partial differential equations as a guiding principle. These equations, the result of a reformulation of classical mechanics, are solved by a scalar function whose gradient describes a continuum of independent classical trajectories which differ by their initial conditions – sort of a wave function without interference. Hamilton had mainly been interested in the elegant mathematical form of this theory rather than in applications. This turned out to be an advantage for Schrödinger. He assumed that Hamilton's equations were no more than a short wave lengths approximation (corresponding to the limit $h \to 0$) of a *fundamental* wave theory – similar to the approximation of geometric optics in Maxwell's theory. However, the short wave length approximation would only mean that local parts of an extended wave propagate almost independently of one another along spatial paths – not that they represent particles. Similarly, Feynman's path integral defines a propagating wave, while it neither requires nor justifies the existence of individual paths that might then simply be selected by a mere increase of information.[3] If focused, different local partial waves or Feynman paths can interfere with one another (that is, they have coherent physical effects). This means that they exist together as *one reality* (one wave function) rather than merely defining a statistical ensemble of *possibilities*. They could be turned into an ensemble only by some stochastic mechanism that would have partially to replace the deterministic wave dynamics.

While light waves propagate in three-dimensional space, Hamilton's waves must according to their construction exist in the configuration space of all possible classical states of the system under consideration. Therefore, Schrödinger, too, obtained wave functions on (what appears to us classically as) configuration spaces of various dimensions. Later, this turned out indeed to be the only empirically correct version of wave mechanics. The intuition of *spatial* wave functions is therefore quite insufficient, in general. The general wave functions can also be understood as a consequence of Dirac's fundamental superposition principle, since the superposition of all classical configurations defines precisely a wave function on configuration space. This concept of a non-spatial wave function can easily be generalized to include properties that never occur as classical variables (such as spin, neutrino flavor, or even the difference between a *K*-meson and its antiparticle), whose superpositions may again define new *individual* physical states (even new kinds of "particles"). In a similar way, Dirac himself understood his superpositions in Born's sense as "probability amplitudes" for properties that are formally represented by Heisenberg's "observables", that is, not only for *points* in configuration space (classical states). There is no absolutely preferred basis in Hilbert space. If these observables are written in terms of dyadic products of their eigenstates (their spectral representation), they may describe Born's probabilities as jumps between wave functions (stochastic projections in Hilbert space as part of the dynamics). Any proposal for some more fundamental theory underlying quantum mechanics would first of all have to explain this very general and well established superposition principle, which, in particular, describes all phenomena of quantum nonlocality without any "spooky" action at a distance (see Sect. 4).

Schrödinger was still convinced of a reality in space and time, and so he initially hoped, in spite of the Hamiltonian analogy, to describe the electron as a spatial *field*. Therefore, he first restricted himself with great success to single-particle problems (quantized mass points, whose configuration space is isomorphic to space). Consequently, he spoke of a "ψ-field". A spatial wave function can also be used to describe scattering problems – either applied to the center-of-mass wave function of an object scattered from a potential, or to the relative coordinates of a two-body problem. In scattering events, Born's probability interpretation is particularly suggestive because of the usual subsequent position measurement in a detector. This wave function in space is in general meant when one speaks of the *wave-particle dualism*. In spite of its limited and therefore misleading value, three-dimensional wave mechanics still dominates large parts of most textbooks because of its success in correctly and simply describing many important single-particle aspects, such as atomic energy spectra and scattering probabilities. It is often supported by presenting the two-slit experiment as *the* key to understanding quantum mechanics, although this is only one specific aspect.

The generalization (or rather the return) to wave functions in configuration space happened almost unnoticed at those times of great confusion – for some physicists even until today. While most of them are now well aware of "quantum nonlocality", they remain used to arguing in terms of spatial waves for many purposes. In contrast to fields, however, even single-particle wave functions do not describe additive (extensive) charge or energy distributions, since each piece cut from a plane wave representing a quantum "particle", for example, would describe its full charge and kinetic energy (if defined by the wave number).

Initially, Schrödinger took great pains to disregard or to re-interpret his general wave equation in configuration space, even though it is precisely its application to oscillating field amplitudes rather than mass points that explains Planck's radiation quanta $h\nu$. (Another early example is the rigid rotator, whose wave function depends on the Euler angles.) The spectrum

$E = nh\nu$ that one obtains for quantum oscillators q_i, which are here given by the amplitudes of fixed field modes that classically oscillate in time with different frequencies ν_i, is proportional to the natural numbers n. Only this specific spectral property admits the concept of additive *energy quanta* $h\nu_i$ – later identified with photons – regardless of any emission process that had often been made responsible for their existence. So it may also explain the concept of "occupation" or "particle" numbers. In Schrödinger's wave mechanics, these quantum numbers n are again explained by the numbers of nodes of the corresponding wave function. The latter has to be distinguished from the considered field mode itself, such as $sin(\boldsymbol{k}_i\boldsymbol{x})$ multiplied with a polarization vector, with its given *spatial* nodes. These field modes (rather than their wave functions) may instead be regarded as "photon wave functions" – see below and Sect. 5.

But where can one find these oscillator wave functions if not in space? In contrast to the figure, they are here defined as functions on the abstract configuration space of field amplitudes q_i. Different eigenmodes of a *classical* field $q(\boldsymbol{x},t)$, such as plane waves with their classical frequencies ν_i, can fortunately be quantized separately; their Hamiltonians commute. This means that energy eigenstates Ψ for the total quantum field factorize in the form $\Psi=\Pi_i\psi_i(q_i)$, while their eigenvalues simply add, $E = \Sigma_i E_i$. Although the oscillator spectrum $E_i = n_i h\nu_i$ can also be derived formally from Heisenberg's algebra of observables (matrix mechanics) without explicitly using wave functions, the latter's nodes for a fixed field mode q_i have recently been made visible for the first time for various "photon number" eigenstates in an elegant though perhaps not yet sufficiently interpreted experiment.[4] The wave functions $\psi_i(q_i)$ on configuration space have thus been shown to *exist*, although they cannot be attributed to the traditional "wave-particle" dualism, which would refer to *spatial* waves characterizing "quantum particles". The importance of this fundamental experiment for the wave-particle debate has in my opinion not yet been appropriately appreciated by the physics community or in textbooks (see Sect. 5 for further details).

The difference between Schrödinger's theory and a classical field theory becomes particularly obvious from the fact that the amplitudes of a classical field now appear as *arguments q* in Schrödinger's wave function. Positions occur here only as an "index" that distinguishes field amplitudes at different space points from one another, as they form a spatial continuum of *coupled* oscillators. Since classical fields are usually written as functions on space and time, $q(\boldsymbol{x},t)$, the confusion of their spatial arguments with dynamical quantum variables (particle positions in quantum mechanics) has led to the questionable concept of a "time operator" for reasons of relativistic space-time symmetry (that cannot be manifest in the canonical formalism). However, $\boldsymbol{x}$ and t are here both classical coordinates, while spacetime distances become dynamical variables only as part of the spatial metric of general relativity – see Sect. 6. While a general time-dependent "one-photon wave function" can be understood as a quantum superposition of various spatial field modes (such as different plane waves) that are in their first excited quantum state ("occupied once" – with all others in their ground state), a quasi-classical *field* state has in QFT to be described as a coherent superposition of many *different* excitations $\psi_i^{(n)}(q_i,t)$ (different "photon numbers" n) for each spatial eigenmode i. In contrast to the free wave packet shown in the figure, these "coherent oscillator states" (time-dependent Gaussians, here as functions of the field amplitude) preserve their shape and width *exactly*, while their centers follow classical trajectories $q_i(t)$. For this reason, they imitate oscillating classical fields much better than wave packets in space may imitate particles.

Field functionals Ψ can thus represent classically quite different concepts, such as "particle" numbers and field amplitudes, mutually restricted by a Fourier theorem. For this reason, the *Boltzmann distribution* $e^{-E/kT}$ of their energy eigenstates may describe the Planck spectrum with its particle and wave limits for short and long wavelengths, respectively. Field functionals can also explain all specific phenomena of quantum optics, such as "bunching".

4. Entanglement and Quantum Measurements

Before trying to study *interacting* quantum fields (see Sect. 5), early quantum physicists successfully investigated the quantum mechanics of non-relativistic many-particle systems, such as multi-electron atoms, molecules and solid bodies. These systems could *approximately* be described by means of different (orthogonal) single-particle wave functions for each electron, while the atomic nuclei seemed to possess fixed or slowly moving positions similar to classical objects. For example, this picture explained the periodic system of the chemical elements. On closer inspection it turned out – at first for atoms and small molecules – that *all* particles forming such objects, including the nuclei, have to be described by one common wave function in their $3N$-dimensional configuration space. This cannot generically be a product or determinant of single-particle wave functions – a consequence that must be extended to all composite systems (including the whole quantum universe), and it became later known as "entanglement". Similar entanglement must in general exist in QFT between the different wave modes q_i when considered as subsystems. David Bohm referred to this property of the wave function as “quantum wholeness” when he began to study its consequences for his theory of 1952, since, generically, the global state does not define wave functions of its subsystems. For this reason, quantum theory can be consistently understood *only* as quantum cosmology (Sect. 6). Historically, the importance of this generic entanglement was either entirely underestimated (or even neglected), or otherwise misunderstood as a mere statistical correlation between subsystems. This misinterpretation was then in turn a major obstacle for the acceptance of an ontic interpretation of the wave function. The presently very popular unitary dynamics of entangled qubits, interrupted by classically described actions of Alice and Bob, is no more than an inconsistent caricature of quantum mechanics.

Every physics student is using the entanglement between an electron and a proton in the hydrogen atom when writing the wave function as a product of functions for center-of-mass and relative coordinates. Nobody would do that for interacting classical *fields*. While the wave function for the relative state defines the size of the hydrogen atom, the center of mass may be represented by a free wave packet, as in Fig. 1. The simplest nontrivial case of entanglement, the Helium atom, was first successfully studied in great numerical detail by Hylleraas, using variational methods, in a series of papers starting in 1929. Already Arnold Sommerfeld noticed in his *Wellenmechanischer Ergänzungsband* that “Heisenberg’s method”, which used only the anti-symmetrization of product wave functions by means of “exchange terms”, is insufficient. (Anti-) symmetrisation is indeed often confused with entanglement, since it is formally equivalent to an entanglement between physical variables and meaningless particle numbers. It merely eliminates any concept of *distinguishability*, and it is therefore not required any more in the occupation number representation (see Sect. 5).† Genuine entanglement in microscopic systems means, for example, that one has to take into account “configuration mixing” as a correction to the independent-particle (Hartree-Fock or mean field) approximation. For long-range interactions, entanglement may be small in the ground state, since according to the independent-particle picture it describes “virtual excitations” (which are often misinterpreted as “fluctuations” rather than static entanglement).

An important consequence of entanglement is that subsystem Hamiltonians are in general not (or not uniquely) defined – thus ruling out *local* unitarity or a uniquely defined Heisenberg or interaction picture for open systems. *Closed* non-relativistic *N*-particle systems, on

† *Separate* (anti-)symmetrization of spin and orbit parts, however, may define *physical* entanglement; in atomic physics, for example, one has to antisymmetrize in jj-coupling in order to avoid it. The statement “particle at position x_1” (in contrast to “particle number 1”) “has spin-up” – as in a Bell type experiment – *is* physically meaningful.

the other hand, have to be described by *one* entangled wave function in their complete configuration space. Their center-of-mass wave functions may then factorize from the rest, thus leading to free *spatial* wave functions for them (identical to those for mass points or "quantum particles"), while the internal energy quantum numbers are given by the number of nodes (or zeros: now defining hypersurfaces) in the remaining $3(N-1)$-dimensional configuration space. For non-inertial motion, this separability is only approximately valid – depending on the required internal excitation energy.[5] Such isolated systems were used to study quantum mechanics, although they represent an exceptional situation. Open system quantum mechanics was introduced much later – mostly in a semi-phenomenological manner, and in combination with statistical physics. Unitary dynamics was rarely applied to global systems in order to *derive* subsystem quantum mechanics, where it later led to the discovery of decoherence.

However, how can the space of all possible classical configurations, which would even possess varying dimensions, replace three-dimensional space as a new fundamental arena for the dynamics of wave functions that may represent physical states? If our Universe consisted of N particles (and nothing else), its configuration space would possess $3N$ dimensions – with N being at least of the order 10^{80}. For early quantum physicists – including Schrödinger, of course – such a wave function was inconceivable, although the concept of a space of *possible configurations* fits excellently with Born's probabilities for classical properties. Entanglement can then conveniently be understood as describing statistical correlations between measured variables. But only between *measured* variables! Since macroscopic variables are "permanently measured" by their environment (see below for decoherence), their entanglement almost always appears to be no more than a statistical correlation. Only this explains why we are used to interpret the space on which the wave function is defined as a "configuration" space. In the mentioned case of the Helium atom, though, entanglement is responsible for the precise energy spectrum and other *individual* properties – regardless of any statistical interpretation. This conceptual difference is often simply "overlooked" in order to keep up the illusion of a merely epistemic interpretation of the wave function (where probabilities would reflect incomplete information about some present or future real properties). Even in individual scattering events one often needs entangled scattering amplitudes with well defined phase relations between all fragments. Only after Einstein, Podolsky and Rosen (EPR) had shown in 1935 that the entanglement between two particles at a distance may have non-trivial observable consequences, did Schrödinger regard this property as the greatest challenge to his theory – although he kept calling it a "statistical correlation". EPR had indeed erroneously concluded from their analysis that quantum mechanics cannot represent a complete description of Nature, so that as yet unknown ("hidden") variables should be expected to exist.

While many physicists speculated that such hypothetical hidden variables might never be observed in an experiment (even though they might exist), it came as a surprise to them when John Bell showed in 1964 that *any* kind of hidden local reality (no matter whether it consists of particles, fields or other local things with local interactions only – observable or not) would be in conflict with certain observable consequences of entangled wave functions. In order to prove this theorem, Bell used arbitrary local variables λ (just a name for something not yet known) for an indirect proof. However, most physicists had by then become so much accustomed to Bohr's denial of a microscopic reality that they immediately accused Bell for having used a "long refuted assumption". The Copenhagen interpretation does indeed clearly go beyond a merely epistemic understanding of the wave function, since, insofar as it refers to ensembles at all, the latter are only meant in a purely formal sense – not in terms of any *elements* (those hidden variables) which would in principle answer the question "Information about what?" In this "operational" approach (supported by Günther Ludwig, for example), the essential problem is therefore simply swept under the carpet.

Crucial direct tests of quantum nonlocality had to be restricted to two- or few-particle systems, which can be isolated from anything else until they are measured. While this nonlocality, a consequence of the superposition principle, has thereby always been confirmed, physicists are still debating whether this fact excludes locality (in three-dimensional space) or any kind of microscopic *reality*. For neither those who accept reality to be described by a nonlocal wave function nor those who deny *any* microscopic reality feel bothered by Bell's theorem. These two camps usually prefer the Schrödinger picture (in terms of wave functions) or the Heisenberg picture (in terms of observables), respectively, and this seems to be the origin of many misunderstandings between them. However, in the absence of any local variables, a locality of interactions (the important "relativistic causality") may appear difficult to define (but see corresponding remarks towards the end of Sect. 5).

If one does assume the superposition principle to apply universally, one is forced to accept one entangled wave function for the whole universe. Heisenberg and Bohr assumed instead that the wave function is no more than a calculational tool, which "loses its meaning" after the final measurement that concludes an experiment. This "end of the experiment" (related to the "Heisenberg cut") remains vaguely defined and *ad hoc*. For example, its traditional application (namely, too early in the chain of interactions that leads to an observation) delayed the discovery of decoherence, which will be discussed below, for many decades. A universal wave function that always evolves according to the Schrödinger equation, however, leads to an entirely novel world view that would be consistent but nonetheless appears inacceptable to most physicists.

For example, if one measures a microscopic object that is initially in a superposition of two or more different values of the measured variable, this gives rise to an entangled state for the microscopic system and the apparatus – the latter including Schrödinger's infamous cat if correspondingly prepared. (All unitary interactions discussed here and below can be assumed to be of a form like $(\sum_n c_n \psi_n)\Phi_0 \rightarrow \sum_n c_n \psi_n \Phi_n$, that is, transforming local superpositions into entanglement – in the "ideal" case without changing or "disturbing" the states ψ_n.) Since such superpositions have never been observed, one traditionally assumes, according to von Neumann, that Schrödinger's dynamics has to be complemented by a stochastic "collapse of the wave function" into one of these components, that is, into a product of narrow wave packets for macroscopic or mesoscopic variables (such as pointer positions Φ_n). Note that, in the Schrödinger picture, Heisenberg's "observables" are readily defined (up to a scale) by this interaction between system and apparatus rather than forming an independent ingredient of the theory. There is then neither a need nor the freedom to *postulate* specific measurements, such as "weak" or "protected" ones, unless used as a unitary interaction. In the Copenhagen interpretation, one would instead pragmatically jump from a description in terms of wave functions to one in classical terms, and back to a new wave function in order to describe a subsequent experiment. This situation is known as the *quantum measurement problem*.

If one is ready, therefore, to accept a universal Schrödinger equation for describing the dynamics of reality, one must try to understand what an entangled wave function for the microscopic system plus an apparatus might mean. Toward that end one has to include the observer into this description.[6] When he reads off the measurement result, he does himself become part of the entanglement. According to the unitary dynamics, he would thereafter simultaneously exist in different states of awareness – similar to the fate of Schrödinger's cat. Hugh Everett first dared to point out in 1957 that this consequence is not in conflict with our subjective observation of *one* individual outcome, since each arising "component state" (or "version") of the observer can register and remember (hence be aware of) only that outcome which is realized in his corresponding "relative state" of the world. The latter would then contain only consistent versions of all observer's "friends" – thus defining objectivized outcomes. As there are many such correlated component states in one *global* superposition, though, the question which of them contains the "true" successor of the physicist who prepared the experiment has no unique answer; according to the unitary dynamics they all do.

However, why can these components be regarded as separate "worlds" with separate observers? The answer is that they are dynamically "autonomous" after a measurement in spite of their common origin; each of them describes a quasi-classical world for its macroscopic variables (see the discussion of decoherence below). In contrast to identical twins, who also have one common causal root, these different versions of the "same" observer cannot even communicate any more according to the unitary dynamics, and thus can conclude each others existence only by means of the dynamical laws they may happen to know. This is certainly an unconventional, but at least a consistent picture, and a straightforward consequence of the Schrödinger equation. It only requires an unconventional identification of states of individual (subjective) observers that is consistent with a nonlocal wave function under local interactions. Attempts to avoid this conclusion are all based on traditional expectations, and lead back to an unsolved measurement problem.

Until recently one preferred to believe, instead, that some conceptual or dynamical border line between micro- and macrophysics must exist – even though it could never be located in an experiment. Otherwise it should be possible (so it seemed) to observe individual consequences of entanglement between microscopic systems and their macroscopic measurement instruments – similar to the energy or other properties of Hylleraas's entangled Helium atom or of small molecules. However, this bipartite entanglement is not yet complete. Macroscopic systems must inevitably, extremely fast, and in practice irreversibly interact with their natural "environment", whereby the entanglement that had resulted from the measurement proper would uncontrollably spread to include much of the "rest of the universe". This happens even before an observer possibly enters the scene. In this way, one may understand how a superposition that extends over different macroscopic pointer positions, for example, would, from the point of view of a potential local observer, inevitably be transformed into an effective ensemble of narrow wave packets that mimic classical states (points in configuration space) as potential outcomes. While still forming one superposition, all these partial waves, which must each include different versions of all observer's "friends", have no chance to meet again in high-dimensional configuration space in order to have local coherent consequences. In this sense only, they can now be *regarded as forming an ensemble* of different "worlds".

This unavoidable entanglement with the environment (whose onset defines the true border line between micro- and macrophysics) is called decoherence,[7] as predominantly phase relations defining certain quantum mechanical superpositions become unavailable – that is, they are irreversibly "dislocalized".[‡] As Erich Joos and I once formulated it, the superposition still exists, but it "is not there" (somewhere) any more. Decoherence is in general a very drastic consequence of quantum dynamics, which requires (and allows) precise numerical calculations only for some mesoscopic systems, such as chiral molecules.[8] For examaple, some of the latter are found in chiral states on Earth, while in parity eigenstates in interstellar space, where the environment is different.

The time asymmetry of the decoherence process requires a specific cosmic initial condition for the wave function, related to that for the thermodynamic arrow of time.[9] However, without Everett's conclusion of splitting local observers ("many minds") as the other nontrivial consequence of universal unitarity, decoherence would not be able to explain the observation of *individual* measurement outcomes. You cannot have one without the other if unitarity is generally valid. Decoherence by itself has therefore occasionally been claimed to be insufficient to solve the quantum measurement problem, but the subsequent splitting of subjective observers amounts for the latter essentially to what Pauli once called the "creation of measurement results outside the laws of Nature"; it is now described as a dynamical consequence of global unitary dynamics *on the observer himself*. Pauli (just as all physicists at his time) simply did not properly take into account the environment and the role of the observer in a consistent quantum setting, that is, in a deeply entangled world.

The experimental demonstration of decoherence as a smooth (though very fast) dynamical process has confirmed that the concept of entanglement does indeed apply beyond

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‡ A mere phase *randomization* ("dephasing") could neither occur under unitary evolution, nor would it solve the issue, as each individual member of an ensemble of local superpositions with different phases would remain a superposition (though possibly with *unknown* phase). Similarly, local phases that are assumed to fluctuate rapidly in time for some reason are in a definite superposition at any instant. Nonetheless, phase averaging forms the most popular misunderstanding of decoherence, which describes entanglement with an uncontrollable environment in the *individual* case (no averaging). These different concepts are easily confused, in particular, if the environment is described as a "thermal bath". However, if this initial thermal "mixture" had been caused by earlier *quantum* interactions with a further environment (which is its most plausible origin), the thus pre-existing entanglement would simply be dynamically extended to the "dephased" variables, where it would then indeed lead to their genuine decoherence (a dislocalization of their *individual* relative phases). Using the reduced density matrix formalism for this purpose would instead tacitly replace nonlocal entanglement by local ensembles. It is remarkable that many important physicists are still missing the essential point of decoherence as a consequence of the fundamental nonlocality of quantum *states*. Nonlocal phase relations may even be required to define observable individual properties (such as the total spin of two particles at very different positions) in spite of contributing to decoherence. – Historically, the term "decoherence" was first invented in the context of "decoherent histories" in about 1985, where it was *postulated* in order to justify "consistent histories" within a conventional probability interpretation, whereas my approach of 1970 was based on an assumed universal unitarity in an attempt to *resolve* the measurement problem – not to tolerate it. Ironically, it is precisely this consequence of universal unitarity that had led to the traditional prejudice that quantum theory does *not* apply to the macroscopic world.

microscopic systems. While this process must remain uncontrollable in order to be irreversible ("real" rather than "virtual"), it has many obvious and important consequences – including apparent quantum jumps and the classical appearance of the world. It also explains why we seem to observe individual atoms as apparent particles in a Paul trap, or tracks in a Wilson chamber as apparent particle trajectories (both are correctly described in terms of narrow wave packets), and why one finds bound microscopic systems preferentially in their energy eigenstates.[7,10] It also allows us to understand the mysterious concept of "complementarity" simply by the different entanglement of microscopic objects with the environment, caused by means of different measurement instruments. This choice of "complementary measurement devices" is not available for systems, such as macroscopic ones, that are already strongly entangled with their environment without being measured by a physicist. The basis "preferred" by a normal environment defines a quasi-classical *configuration* space for such systems, which include even major parts (such as human neural systems) of the thus partially classical observers. While *virtual* decoherence had always been known in the form of microscopic (reversible and often even usable) entanglement, the unavoidable and irreversible effect of the environment on macroscopic systems was overlooked for five decades, mainly because quantum mechanics was traditionally assumed *not* to apply beyond microscopic systems. Surprisingly, the apparently reversible dynamics of classical mechanics does in quantum mechanics require the permanent (though mostly thermodynamically negligible) action of irreversible decoherence.

In order to illustrate the enormous number of new "worlds" that are permanently created according to decoherence (or must otherwise be permanently annihilated by a collapse mechanism), let me consider the example of a two-slit experiment. Measuring which slit the "particle" passes would about double the number of worlds, but registration of the particle on the second screen causes a multiplication of worlds by a large factor that depends on the remaining coherence lengths for the positions of the arising spots. (Everett "worlds" are not *exactly* defined, and thus cannot simply be counted; they may even form an overcomplete set.) This definition of branch worlds by their irreversible separation in configuration space means also that quantum computers do *not* simultaneously calculate in parallel worlds (as sometimes claimed) if they are to produce a coherent result that may then be used in "our" world, for example; "real" (rather than virtual) branches never recohere to form local superpositions again.

Most "particles" in the two-slit experiment do not even pass the slits, but are absorbed on the first screen. This absorption corresponds to a position measurement, too – regardless of whether its information is ever extracted. In order to cause decoherence, this “information” may even be thermalized (erased in the usual sense). In contrast, a “quantum eraser” requires a local superposition to be *restored*, that is, re-localized, rather than information to be destroyed, as its inappropriate name may suggest. Similar considerations apply to most scattering events between “particles” or between other objects and their environments. For M such measurement-like events in the past history of the universe with, on average, N effective outcomes, one would obtain the huge number of N^M now existing branches. Nonetheless, the global configuration space remains almost empty because of its high dimension; the myriads of branching wave packets that have ever been created by *real* decoherence describe separate “worlds” for all reasonable times to come. Nobody can calculate such a global wave function, but under appropriate (far from equilibrium) initial conditions for the universe, its unitary dynamics can be used consistently to justify (1) quasi-classical properties and behavior for all degrees of freedom that are “robust” against decoherence, (2) statistical methods (retarded probabilistic master equations) for most others,[9] and (3) individual wave functions for appropriately prepared microscopic systems. In the case of controllable non-local entanglement, this latter kind of preparation can even be applied at a distance – a phenomenon known as “quantum steering”. These three dynamical applications are then also sufficient to construct measurement devices to begin with. No phenomenological concepts (such as particles, events, pointer positions, or even Alice and Bob) are required on a *fundamental* level.

The observation of radioactive decay represents another measurement of a continuous variable (namely, the decay time). Its precision cannot be better than the remaining coherence time (which is usually very much smaller than the half-life, and thus gives rise to apparent quantum jumps). This coherence time depends on the efficiency of the interaction of the decay fragments with their environment, and it would be further reduced by permanent registration of the (non-) decay. If an excited state decays only by emission of weakly interacting photons, however, decoherence may be slow. One may then even observe coherence between different decay times, thus definitely excluding genuine quantum jumps (“events”) in this case. There is no reason to assume that this would be different if the photon had travelled astronomical distances before a coherent state vector revival, for example.

Many leading physicists who are not happy any more with the Copenhagen interpretation nonetheless prefer to speculate about some novel kind of dynamics (an as yet unknown collapse mechanism) that would avoid the consequence of Many Worlds. This is at present no more than prejudice combined with wishful thinking, but it could in principle also solve the measurement problem in terms of an ontic (in this case partially localized) universal wave function without requiring Everett's multiple observers. One should keep in mind, though, that all as yet *observed* apparent deviations from unitarity, such as quantum jumps or measurements, can be well described (and have in several cases been confirmed experimentally) as smooth decoherence processes in accordance with a global Schrödinger equation. Therefore, if a genuine collapse mechanism did exist after all, it would presumably have to be *triggered* by decoherence, but it could then hardly have any observable consequences on its own.

For example, if one of two spatially separated but entangled microscopic systems (such as those forming a "Bell state") was measured, their total state would according to a unitary description become entangled with the apparatus, too, and thus also with the latter's environment. While this process leads to the formation of dynamically autonomous branches, an observer at the location of the second system, say, becomes part of this entanglement (and therefore "splits") only when he receives a signal about the result. Before this happens, his state factors out, and he may be said not yet to *know* the result. If he then also measured the second system (that at his own location in this case), the state of his memory must thereafter depend on the outcomes of both measurements, that is, it must have split twice unless there was an exact correlation between the results. Since the order of these two measurements does not matter, in general, this description includes delayed choice experiments. In contrast, a genuine collapse caused by the measurement would have to *affect* distant objects instantaneously (whatever that means relativistically) in order to avoid other weird consequences. This *would* then define the "spooky" part of the story.

In this way, an apparent ensemble of quasi-classical "worlds" is for all practical purposes ("FAPP") sufficiently defined by the autonomous branches of the wave function that arise from decoherence: a measurement cannot be undone in practice as soon as the global superposition cannot be re-localized in configuration space, while reasonable observer states can then only evolve separately within different branches. However, neither can we at present *exactly* define conscious observer systems in physical terms, nor does this mechanism explain Born's rule, since all members of the apparent ensemble remain part of one superposition (the "bird's perspective"). Many of them would, in series of measurements, even describe frequencies of outcomes that are *not* in accord with Born's rule. What we still need, therefore, is a probabilistic characterization of the permanently branching quasi-classical world in which "we" happen to live after a measurement.

In all interpretations of quantum mechanics, Born's rule has to be *postulated* (in addition to the unitary dynamics) on empirical grounds in some form. In principle, this remains true in the Everett interpretation, too, but the situation is now partly solved by decoherence, as the members of an *effective ensemble* of outcomes (namely, the dynamically autonomous branches) have been sufficiently defined. (In contrast to the splitting observers, the branching of "worlds" is not a fundamental concept; it is a dynamical consequence of unitarity.) According to their definition by robustness against decoherence, there are no autonomous branches that contain Schrödinger cats or sugar molecules in parity eigenstates, and the like: their probability is zero. All we still have to postulate for the remaining branches are subjective probability *weights*, which should not be affected by the imprecise and time-dependent definition of the branches. The only appropriate candidate for them is the squared norm (their *formal* measure of size), as it is additive and conserved under the unitary dynamics. It is thus not affected by any subsequent finer branching. (For example, further branching occurs during subsequent physical information processing, such as photon "measurements" on the retina, or by measurement-type events somewhere else in the universe.) Therefore, this property gives rise to individual probabilities for apparent collapse events, and thus to the concept of "consistent histories". Everett regarded this dynamical argument, which is similar to the choice of phase space volume as a probability measure in classical statistical mechanics, as *proof* of Born's probabilities.[11] However, only *after postulating* them, does the concept of a density matrix (called a "mixed state") become justified as a tool for calculating expectation values.

By consistently using this global unitary description, all those much discussed "absurdities" of quantum theory can be explained. It is in fact precisely how they were all predicted – except that the chain of unitary interactions is usually cut off *ad hoc* by a collapse at the last relevant measurement in an experiment, where the corresponding decoherence defines a consistent position for the hypothetical Heisenberg cut. Therefore, all those "weird" quantum phenomena observed during the last 80 years can only have surprised those who had never accepted a universal validity of the quantum formalism. Absurdities, such as "interaction-free measurements", arise instead if one assumes the quasi-classical phenomena (such as apparent events) rather than the complete wave function to describe "reality". If the wave function itself represents reality, however, any post-selected component cannot describe the previously documented past any more, which would have to be the case if this post-selection were no more than an increase of information.

So-called quantum teleportation is another example where one can easily show, using unitary dynamics, that nothing is ever "teleported" that, or whose deterministic predecessor, was not prepared *in advance* at its intended position in one or more components of an entangled initial wave function.[10] This confirms again that nonlocal wave functions cannot merely represent a bookkeeping device – even though a local observer *may assume* that an objective global collapse into a non-predictable outcome had already occurred (or that this outcome had been *created* in some other kind of "event") as a consequence of the first irreversible decoherence process during a measurement. It is precisely this possibility that justifies the usual pragmatic approach to quantum mechanics (including the Copenhagen interpretation or von Neumann's collapse during a measurement). However, if one assumed only local properties, such as measurement outcomes, to describe reality, one would indeed have to believe in teleportation and other kinds of spooky action at a distance. According to the Everett interpretation, the usual restriction of "our" quantum world to a tiny and permanently further collapsing *effective* wave function therefore represents no more than a pragmatic convention reflecting the observer's changing situation rather than a physical process. Such a "collapse by convention" may even be assumed to apply instantaneously (superluminally), but it should be evident that a mere convention cannot be used for sending signals.

If the global state does indeed obey unitary dynamics, the observed quantum indeterminism can clearly *not* reflect any objective dynamical law. In Everett's interpretation, it is in principle a "subjective" phenomenon, based on the branching histories of all conceivable observers into many different versions ("many minds"). This may *explain* Heisenberg's interpretation of quantum measurements as requiring "human" observers. This passive indeterminism nonetheless allows observers to *prepare* pure states of microscopic systems in the laboratory as initial conditions for subsequent experiments by selecting the required outcomes in appropriately designed series of measurements. All measurement outcomes are objectivized by the correlation between those versions of *different* observers (including Wigner's friend or Schrödinger's cat) who exist in the same Everett branch, and thus can communicate with one another. For all practical purposes, their entanglement with the apparatus after reading it, and with the environment, also justifies Bohr's interpretation of measurements (unlike Heisenberg's) in terms of classical outcomes that would be irreversibly and objectively created (in apparent events) by the macroscopic apparatus. This macroscopic entanglement (essentially including decoherence) explains the traditional concept of a "classical reality": only a documented phenomenon is a phenomenon (see Footnote **). However, only if the resulting global superposition was misinterpreted as a *statistical ensemble* consisting of the autonomous branches, would an observation of the outcome appear as a mere increase of information.

5. Quantum Field Theory

We have seen that quantum mechanics in terms of a universal wave function admits a consistent (even though novel kind of) description of Nature, but this does not yet bring the strange story of particles and waves to an end. Instead of *spatial* waves (fields) we were led to wave functions on a high-dimensional "configuration space" (a name that is justified only because of its appearance as a space of *potential* classical states according to decoherence). For a universe consisting of N particles, this configuration space would possess $3N$ dimensions, but we may conclude from the arguments in Sect. 3 that for QED (quantum electrodynamics) it must be supplemented by the infinite-dimensional configuration space of the Maxwell fields (or their vector potentials in the canonical formalism). A product of wave functions for the amplitudes of all field modes in a cavity or in free space turned out to be sufficient to explain Planck's quanta by the number of nodes of these wave functions. The spontaneous occurrence of photons as apparent particles (in the form of clicking counters, for example) is then merely a consequence of fast decoherence of the entangled state in the detector.

However, we know from the quantum theory of relativistic electrons that they, too, have to be described by a *quantized field* (that is, by a field functional) – a consequence that must then also apply to the non-relativistic limit. There are no particles even *before* quantization any more. The relativistic generalization of a one-electron wave function is called the *Dirac field*, since it is usually studied as a function on spacetime. Dirac proposed it at a time when Schrödinger's wave function was mostly believed to define a spatial field for each electron, but the Dirac field can *not* be generalized to an N-electron field on a $4N$-dimensional "configuration spacetime", although this has occasionally been proposed; there is only one time parameter describing the dynamics for the total state. In the Schrödinger picture of QED, the Dirac field is used to define, by its configuration space and that of the Maxwell field, the space on which the corresponding time-dependent wave functionals live. According to the rules of canonical quantization, these wave functionals have to obey a generalized Schrödinger equation again (the Tomonaga equation).[12] Spin and other internal degrees of freedom thereby become part of the "classical" (non-quantized) fields.

This consequence of QFT avoids a fundamental N-dependence of the relevant configuration spaces for varying numbers N of "particles", as it allows for a concept of "particle creation", such as by raising the number of nodes of the field functional (cf. Sect. 3). Relativistic covariance cannot and need not be manifest in this canonical formalism. For example, the canonical quantization of the Maxwell field leads consistently to a wave functional $\Psi\{\boldsymbol{A}(\boldsymbol{x});t\}$, with a vector field $\boldsymbol{A}$ defined at all space-points $\boldsymbol{x}$ on an arbitrary simultaneity t. Since Schrödinger had originally discovered his one-electron wave function by the same canonical quantization procedure (applied to a single mass point), the quantization of the Dirac field is for this purely historical reason also called a "second quantization". As explained above, though, the particle concept, and with it the first quantization, are no more than historical artifacts.[13]

Freeman Dyson's "equivalence" of using relativistic field functionals (Tomonaga) or field operators (Feynman)[14] is essentially based on the (incomplete) equivalence between the Schrödinger and the Heisenberg picture. However, the Heisenberg picture would hardly be able even in principle to describe the hefty, steadily growing entanglement required by a time-dependent global wave function. Since relativity is based on the absence of absolute simultaneities, the relativistic generalization of the Schrödinger equation can indeed only be given by the Tomonaga equation with its "many-fingered" concept of time (arbitrary simultaneities). Apparent particle lines in Feynman diagrams, on the other hand, are merely shorthand for certain field modes (such as plane waves, with "particle momenta" representing their wave numbers).[3] These diagrams are used only as intuitive tools to construct terms of a perturbation series in terms of integrals over products of such field modes and other factors, mainly for calculating scattering amplitudes. In this picture, closed lines ("virtual particles") describe entanglement between quantum fields. Since high-energy physics is mostly restricted to scattering experiments, unitarity is in many textbooks quite insufficiently explained as describing the "conservation of probability" – thus neglecting its essential consequence for the quantum phases, which are needed to determine general superpositions arising in a scattering process.

The Hamiltonian form of the Dirac equation is unusual because of its linearization in terms of particle momentum: the classical canonical momenta are not given by time derivatives of the position variables (velocities) any more. Nonetheless, the two occupation numbers 0 and 1 resulting from the *assumption* of anti-commuting field operators[§] are again interpreted

[§] Let me emphasize, though, that the origin of the Pauli principle, which is valid for fermions, does not seem to be entirely understood yet. While the individual components of the Dirac spinor also obey the Klein-Gordon equation, the latter's quantization as a field of coupled oscillators would again require *all* oscillator quantum numbers $n = 0,1,2,\ldots$. Anti-commuting field operators, which lead to anti-symmetric multi-particle wave funcas “particle” numbers because of their consequences in the quasi-classical world. Field modes “occupied” once in this sense and their superpositions define again “single-particle wave functions”. In contrast to the case of photons, however, one never observes superpositions (wave functionals) of *different* electron numbers. This has traditionally been regarded as a fundamental restriction of the superposition principle (an axiomatic “superselection rule”), but it may be understood as a (here kinematical) consequence of decoherence: for charged particles, their Coulomb field assumes the role of an environment.[15]

In QFT, the formulation that one particle is in a quantum state described by the spatial wave function ψ_1, and a second one in ψ_2, is thus replaced by the statement that two *field modes*, ψ_1 and ψ_2, are both in their first excited quantum state (“occupied once”). A permutation of the two modes does not change this statement that is based on a logical “and”, so there is only *one* state to be counted statistically. This eliminates Gibbs’ paradox in a very natural way. (Schrödinger seems to have used a similar argument in favor of waves instead of particles even before he explicitly formulated his wave equation.[16])

It would similarly be inappropriate to claim that *wave functions* can be directly observed in Bose-Einstein condensates (as is often done). What one does observe in this case are again the (now multiply “occupied”) three-dimensional boson *field modes* – even though massive bosons are traditionally regarded as particles. Instead of the *free* field modes used for photons, however, interacting bosons are more appropriately described in terms of self-consistent field modes in analogy to the self-consistent Hartree-Fock single-fermion wave functions. Both cases lead to what is regarded as an effective non-linear “single-particle wave

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tions, were postulated quite *ad hoc* by Jordan and Wigner, and initially appeared artificial even to Dirac. Interpreted rigorously, their underlying configuration space (defining a Hilbert space basis again) would consist of a spatial continuum of coupled bits (“empty” or “occupied”) rather than a continuum of coupled oscillators. The n-th excited state of this bit continuum (that is, n occupied positions) represents *n identical* point-like “objects”. Because of the dynamical coupling between bit-neighbors, these objects can move, but only *after* their quantization, which leads to entangled superpositions of different occupied space points, may they give rise to propagating waves. In order to be compatible with this bit continuum, their coefficients (“multi-particle wave functions”) must vanish whenever two of their arguments coincide. This can quite generally be achieved by assuming them to be antisymmetric under permutations of any two arguments. No field algebra is explicitly required for this argument (although it could then be consistently defined). In this picture, single-fermion wave functions would represent genuine quantum states (quantum superpositions) rather than wave modes as for bosons. In contrast, coupled oscillators defining a free boson field propagate as spatial waves, and thus obey a *classical* superposition principle (in space rather than in their configuration space) in addition to the quantum superposition principle that is realized for them by the field functionals. This difference is particularly dramatic in Bohm’s theory, where one often meets disagreement on whether its trajectories have to include photons as particles or a time-dependent vector potential (a classical field). However, these pre-quantization concepts need not possess any physical meaning by themselves. Moreover, a fundamental distinction between bosons and fermions may be problematic for *composite* “particles” (dressed fields).

equation" – for bosons called the Gross-Pitaevskii equation.** In spite of this effective non-linearity, the quantum states proper are, of course, described by the linear Schrödinger equation – relativistically always in the sense of Tomonaga.[12]

As already mentioned in Sect. 3, photon number eigenfunctions $\psi^{(n)}(q)$ in the configuration space of wave amplitudes q – to be distinguished from their three-dimensional field modes ("single-photon wave functions", which are fixed modes in a cavity in this case) – have recently been confirmed for various values of the "particle number" n by means of their Wigner functions to be real and observable.[4] For pure states, Wigner functions are defined as partial Fourier transforms of the dyadic products $\psi^{(n)}(q)\psi^{(n)*}(q')$, and thus equivalent to the wave functions $\psi^{(n)}(q)$ themselves (except for a total phase). The variable q is here the amplitude of the given field mode rather than some spatial position as in single-particle quantum mechanics. The two-dimensional Wigner functions on their apparent phase space q,p were made visible in this experiment, and so allow one to clearly recognize the n nodes of the wave functions $\psi^{(n)}(q)$ (forming circles in phase space). Creation and annihilation operators are defined to change the number of these nodes. Since they occur dynamically only in the Schrödinger equation, they describe *smooth* physical processes (time-dependent wave functionals), while creation "events" are either meant just conceptually, or would require a fast decoherence process. The *physical* nature of the field functionals is also confirmed by their ability to participate in general entanglement and, in this way, contribute to the observable decoherence of their sources without having to affect any absorbing matter.

** At normal temperatures, "many-particle" systems (that is, multiple quantum field exitations) may behave approximately like a gas of classical particles undergoing stochastic collisions because of their permanent mutual decoherence into apparent ensembles of narrow spatial wave packets for the individual "particles".[28] This consequence perfectly justifies Boltzmann's *Stosszahlansatz* – but *not* any quasi-deterministic particle trajectories. The concept of trajectories would approximately apply only for heavy objects that suffer mainly "pure" decoherence (with negligible recoil). "Open" quantum systems are generally described by similar phenomenological (Lindblad-type) master equations that are usually *postulated* rather than being derived from realistic assumptions for a quantum environment, and often misunderstood as representing fundamental deviations from unitary quantum mechanics. In order to be regarded as "macroscopic" in the sense of "always given" or "known", that is, not part of a thermal distribution, quasi-classical (decohered) variables have furthermore to be redundantly documented in the rest of the universe (see under "fork of causality", "consistency of documents", or "overdetermination of the past" in the first Ref. 7 – for example in Footnote 1 on its page 18). Dynamically conserved "information" *about* such systems may nonetheless be dynamically exchanged between microscopic and macroscopic variables. – In the theory of "quantum Darwinism",[29] these *classical* thermodynamic arguments are intermingled (and perhaps a bit confused) with the quantum concept of decoherence, which represents spreading physical entanglement, but not necessarily any spreading of (usable) information into the environment. Transfer of (necessarily physical) information must always cause decoherence, but the opposite is *not* true: even an environment in thermal equilibrium may allow the formation of further entanglement with a "system" under consideration. Documents which define *humanistic history* – including the history of science – obviously require even more specific correlations than those responsible for thermodynamic macroscopicity only.

For relativistic reasons, *all* known elementary physical objects are described as quantum fields (although they are usually called "elementary particles"). The contrast between the first order in time of the Schrödinger equation and the second order of classical field equations with their negative frequencies opens the door to the concept of "anti-bosons". (For fermions this relation assumes a different form – depending on the starting point before quantization, as indicated in Footnote §.) Because of the universality of the concept of quantum fields, one may also expect a "theory of everything" to exist in the form of a unified quantum field theory. At present, though, the assumption that the fundamental arena for the universal wave function be given by the configuration space of some fundamental field(s) is no more than the most plausible attempt. On the other hand, the general framework of Schrödinger's wave function(al) or Dirac's superposition principle as a universal concept for quantum states which obey unitary dynamics has always been confirmed, while attempts to derive this framework from some deeper ("hidden") level have failed so far, and are strongly restricted by various no-go theorems.

Among boson fields, gauge fields play a surprisingly physical role, since gauge transformations appear locally as unphysical redundancies. Their physical role is facilitated by their dynamical entanglement, which thus reveals that the redundancy holds only classically. Gauge variables then appear as purely relational quantities.[17] An important question after quantization is whether gauge symmetries can be broken by a real or apparent collapse.

Unfortunately, *interacting* fields in general require the entanglement of such an enormous number of fundamental degrees of freedom – traditionally interpreted as "quantum fluctuations" even in time-independent states – that they cannot even approximately be treated beyond a questionable (though often successful) perturbation theory in terms of free effective fields. Instead of consistently applying the established concepts of quantum mechanics (namely, entangled superpositions) to the new variables (such as field amplitudes), various semi-phenomenological concepts are therefore used for specific purposes in QFT – mostly for calculating scattering amplitudes between objects that are assumed to be asymptotically free (which never happens for macroscopic objects, though). This is not very different from the long-time belief that quantum mechanics does not apply to macroscopic systems. *Stable* local entanglement can here be regarded as a "dressing" of fields (similar to the entanglement between proton and electron in the bound hydrogen atom – cf. Sect. 4), while chaotic and nonlocal entanglement defines decoherence. Only for individual field modes, as in cavity QED, one may explicitly investigate their entanglement, for example with individual atoms.

Even these semi-phenomenological methods are severely haunted by infinities resulting from local products of field operators that are assumed to appear in the effective Hamiltonians. The construction and interpretation of these methods is mostly based on particle concepts again (such as in Feynman diagrams, or by interpreting clicks and bubbles appearing in detectors as being caused by particles). Therefore, "effective" quantum fields cannot be expected to represent fundamental variables that might be revealed by mere "renormalization" procedures. This opens up quite novel possibilities, perhaps even to understand all fermions as quantum consequences of certain topological defects (such as superpositions of different locations of topological singularities – cf. Footnote § again). In other words: we know almost nothing about a fundamental theory.

Similar semi-phenomenological methods as in QFT are also used in condensed matter physics, even when its objects of interest are non-relativistically regarded as N-particle systems. They may nonetheless give rise to effective phonon fields or various kinds of "quasi-particles". The wave function for the lattice ions and their electrons, for example, is here regarded as fundamental, while the phonon field functional "emerges" – similar to Goldstone bosons in QFT. Symmetry-breaking effective ground states (such as lattices with fixed positions and orientations) and their corresponding "Fock spaces" can be understood as representing Everett branches that have become autonomous by the decoherence of their superpositions into wave packets that must have accompanied their condensation process.[18] Some such "Fock vacua" are characterized by the number of certain particles (such as electrons in a metal) that form a *stable entanglement* in this ground state. Most familiar are pair correlations in the BCS model of superconductivity. A similar model in QFT led to the prediction of the Higgs "particle". However, only in situations described by an effective Hamiltonian that gives rise to an energy gap (defining an effective mass) can the lowest excited states approximately avoid further decoherence within their corresponding Fock space under normal conditions and low temperatures, and thus exhibit the usual phenomena of "single particle" quantum mechanics (Sect. 4).

The BCS (pair correlation) model is also useful for understanding the origin of Hawking and Unruh radiation,[19] which are often misinterpreted as representing vacuum fluctuations rather than entanglement. Since only exceptional field modes of a given space volume would obey boundary conditions also for its subvolumes, not even the total ground state factorizes into *local* subvacua. The Hilbert space Hamiltonian, therefore, depends not only on the differential operator, but also on the chosen boundary conditions (which define its eigenstates), while complementary subvolumes are entangled in almost all pure *total* states in QFT. For parallel *physical* boundaries, which require a singular energy renormalization by the zero-point energies of all thereby excluded field modes, this leads to the Casimir effect as a measurable (finite) dependence on distance between the plates. Without physical boundaries, the entanglement may be regarded as a *static* mutual decoherence of the open subvolumes. Non-inertial detectors, for example, define real or apparent spacetime horizons as formal boundaries for the modes to which they couple, and thus register an (improper) thermal mixture representing Hawking or Unruh radiation in the inertial vacuum (which extends beyond these horizons). The presence of "particles" (field excitations) is here a matter of spacetime perspective, based on the choice of non-inertial reference frames that are used to define "plane" waves as relevant field modes, while general quantum *states*, such as various "physical vacua", are *objectively* defined by their physical (such as cosmological) boundary conditions – and thus represent "real" states in spite of their ambiguous interpretation in terms of particles.

In *microscopic* many-particle systems, for example in small molecules or atomic nuclei, spontaneous *intrinsic* symmetry breaking may even lead to energy eigenstates for collective motions (such as rotations or vibrations). Since electrically neutral microscopic objects can often be assumed to be isolated from their environments, asymmetric "model ground states" (deformed nuclei or asymmetric, such as chiral, molecular configurations) give rise to degeneracy and thus to energy bands or multiplets by means of different superpositions of all their degenerate orientations or chiralities.[20] The corresponding collective degrees of freedom are often classically visualized as describing slow ("adiabatic") motion, although this would in turn require time-dependent superpositions of *different* energy eigenstates. The quantum mechanical justification of such time-dependent states, which are observed for *macroscopic* objects, had to await the discovery of decoherence (here of the energy eigenstates). Since all particles in a collective superposition of different orientations are strongly entangled with one another, their energy eigenstates are analogous to the bird's perspective of a quantum world, while an external observer of such a state assumes the role of a "real bird" in this analogy. In contrast, the whole quantum world must *contain* its observer, who thus gives rise to "many minds" with their asymmetric frog's perspectives (broken symmetries).[18] In accordance with this picture, individual particles contributing to collective rotational superpositions are known in first approximation to feel only a *fixed* deformed potential (analogous to observing a definite measurement outcome), as can be seen from their single-particle spectra (for example Nielson states in deformed nuclei – a variant of the nuclear shell model). This fact was the starting point for the many-minds interpretation. In this sense, collective superpositions imitate a "multiverse" consisting of different orientations, but these quantum cosmological analogies seem to have delayed the acceptance of the concept of decoherence for a decade, until its "naïve" (circular) interpretation by means of the pragmatically justified reduced density matrix formalism became popular and made it acceptable to practicing physicists. In the case of a *global* symmetry, collective variables bear some similarity to gauge variables.

On a very elementary level, semi-phenomenological methods were already used for the hydrogen molecule by separately quantizing its "effective" degrees of freedom (center of mass motion, vibration, rotation and two independent electrons in the Coulomb field of adiabatically moving nuclei) rather than treating it exactly as an entangled four-body problem. Chiral molecules can at very low energies effectively be described as two-state systems, while an analogous explanation may conceivably await discovery for *all* kinds of qubits.

In QFT, the successful phenomenology of apparently fundamental fields ("elementary particles"), such as described by the Standard Model, may be expected to form the major touchstone for any fundamental theory of the future. This may be true even though quantum chromodynamics seems to be already too complex for us to derive nuclear physics phenomena without auxiliary assumptions. At present, this Standard Model, which basically consists of linear representations of phenomenological symmetry groups whose physical meaning is not yet understood, does not seem to offer any convincing hints for the precise nature of an elusive fundamental theory. All one may dare to predict is that its Hilbert space must possess a local *basis* (such as the configuration space of spatial fields of local properties and/or point-like objects) in order to allow for a definition of dynamical locality or "relativistic causality". In contrast to popular concepts of *mono*-causality, however, classical reality is multi-causal: in order to determine the fields at some spacetime point in classical field theory, one has to know them on a complete slice through its (past or future) light cone. Only since the causal connection between two *events* may then be difficult to check, did Einstein postulate the travel of "signals", which may be characterized by some identifyable structure, rather than general causal influences, to be limited by the speed of light. Even though quantum superpositions of local fields are kinematically nonlocal, and thus able to violate Bell's inequality, the *dynamical* locality defined for their basis remains effective for them (including interactions that describe measurements and decoherence).[21] In particular, local interactions can cause entanglement only between subsystems on their future light cones. While nonlocal phase relations, defining superpositions, are essential for the precise value of von Neumann's conserved global ensemble entropy (zero for pure states, for example), the dynamical transformation of information ("negentropy") *about* local systems into that about nonlocal correlations or entanglement describes the increase of "physical entropy", since the latter is defined as additive, and thus neglects nonlocal correlations for being thermodynamically "irrelevant".[9]

This search for the Hilbert space basis of a fundamental theory has nothing to do with that for "hidden variables", which are to explain quantum indeterminism and the wave function themselves. All novel theories that are solely based on mathematical arguments, however, have to be regarded as speculative until empirically confirmed – and even as incomplete as long as there is no general consensus about the correct interpretation of their quantization. Many quantum field theorists and mathematical physicists seem to regard their semi-phenomenological models, combined with certain methods of calculation and applied to classical field or particle concepts, as *the* quantum field theory proper. Indeed, why should one expect a consistent theory if there is no microscopic reality to be described – as assumed in the still popular Copenhagen interpretation and its variants? Therefore, most textbooks of QFT do not even *attempt* to present a conceptually consistent and universally well defined theory.

Our conclusion that the observed particle aspect is merely the consequence of fast decoherence processes in the detecting media does not seem to be of particular interest to many high-energy physicists, although quantum phenomena in their detectors are an essential part of their experiments. Some of them call the enigmatic objects of their research "wavicles", as they cannot make up their mind between particles and waves. This indifferent language represents another example for Wigner's "Balkanization of physics" (or "many words instead of many worlds" according to Tegmark). The wave-particle "dualism" is usually still understood with respect to *spatial* waves rather than wave functions in configuration space, although the former should by now be known to be quite insufficient in quantum theory.

6. Quantum Gravity and Quantum Cosmology

I cannot finish this presentation of quantum theory without having mentioned quantum gravity. Although one cannot hope to observe quanta of gravity in the foreseeable future, the formal quantization of gravity can hardly be avoided for consistency in view of the quantization of all other fields. Its dynamical variables must then also appear among the arguments of a universal wave function, and thus be entangled with all other fields – in a very important way, as it turns out.[22]

The Hamiltonian formulation of Einstein's general relativity was brought into a very plausible form by Arnowitt, Deser and Misner in 1962. They demonstrated that the configuration space of gravity can be understood as consisting of the spatial geometries of all possible three-dimensional space-like hypersurfaces in spacetime. These hypersurfaces define arbitrary simultaneities that may form various foliations of spacetime, and which may then be parametrized by a time coordinate t. This Hamiltonian form of the theory is therefore also called "geometrodynamics". Its canonical quantization leads to a (somewhat ambiguously defined) Schrödinger equation in the sense of Tomonaga for the wave functional on all these geometries – known as the *Wheeler-DeWitt equation*. This is another example which demonstrates that the Hamiltonian form of a theory is not in conflict with its relativistic nature.

In contrast to the normal Schrödinger equation, the WDW equation remarkably assumes the form $H\Psi = 0$. It can also be understood as a constraint, while the Schrödinger equation itself then becomes trivial: $\partial\Psi/\partial t = 0$. The reason is that there is no classical spacetime any more to be foliated. (Each foliation of spacetime would correspond to a *trajectory* through this configuration space). However, the spatial metric that occurs (besides matter variables) as an argument of the wave functional Ψ would determine all proper times ("physical times") along time-like curves which connect it classically, according to the Einstein equations, with any other conceivable spatial geometry – regardless of the choice of a foliation. Therefore, in spite of its formal timelessness, the Wheeler-DeWitt equation *does* define a physical time dependence by means of the entanglement between all its variables – similar to the entanglement $\psi(u,q)$ between a clock variable u and other variables q in quantum mechanical description. Therefore, the *formal* timelessness of the WDW equation is a genuine quantum property that reflects the absence of classical trajectories. Classical spacetimes correspond to trajectories that can be parametrizied by a coordinate time t (albeit invariantly reparametrized by monotoneous functions $t'(t)$). *Physical* time is in general many-fingered, that is, it depends on the local progression of the space-like hypersurfaces independently at any space point. In the case of an exactly homogenous and isotropic Friedmann cosmology, it may conveniently be represented by *one* single "finger": the expansion parameter a. If the wave function were regarded as a probability amplitude, it would now define probabilities *for* physical time; it is not a function *of* (some external) time any more.

It is further remarkable in this connection that, for Friedmann type universes, the Hamiltonian constraint $H\Psi = 0$ assumes a hyperbolic form in its infinite-dimensional configuration space – again with a or its logarithm defining a time-like variable. This property is physically very important, since it allows for a global "initial" value problem for the wave functional – for example at $a \rightarrow 0$.[23] For increasing a, its solution may then form a superposition of wave packets that "move" through this configuration space as a function of a. Even a drastic asymmetry of Ψ with respect to a reversal of a (an "intrinsic" arrow of time) might be derivable from symmetric boundary conditions (such as the usual integrability condition in a) because of the asymmetry of the Hamiltonian under this reversal.

Claus Kiefer could furthermore derive the time-dependent Schrödinger (Tomonaga) equation for the matter wave function under a short wave length approximation for the geometric degrees of freedom. It corresponds to a Born-Oppenheimer approximation with respect to the inverse Planck mass (see Kiefer's Ch. 4 in Joos et al. of Ref. 7, or Sect. 5.4 of Ref. 22). This Hamiltonian form emphasizes the fact that the Wheeler-DeWitt equation can only describe a whole Everett multiverse, since each trajectory in the configuration space of spatial geometries would define a (possibly different) classical spacetime. Wave packets for spatial geometry approximately propagating along such trajectories are decohered from one another by the matter variables (which thereby serve as an "environment"). This is analogous to the decoherence of atomic nuclei in large molecules by collisions with external particles – the reason why they appear to move on quasi-classical trajectories according to the frog's perspective of a human observer. In cosmology, decoherence (that is, uncontrollable entanglement rather than the often mentioned "quantum fluctuations") is also important for the origin of "classical" structure in the early universe during the onset of inflation.[24]

If one also allowed for "landscapes" (Tegmark's Level 2 of multiverses[25]), which are assumed to exist in several hypothetical cosmologies that lead to a drastically inhomogeneous universes on the very large scale, the "selection" (by chance – not by free will) of a subjective observer with his epistemologically important frog's perspective (cf. Sect. 4) may be roughly characterized by a hierarchy of five not necessarily independent steps: (1) the selection (in the sense of Level 3, that is, Everett, usually regarded as a quantum measurement) of an individual landscape from their superposition that must be part of a global quantum state, (2) the selection of a particular region in this three or higher dimensional landscape (a causally separate "world" that may be characterized by specific values of certain "constants of nature" – Level 2), (3) the selection of a quasi-classical spacetime from the Wheeler-DeWitt wave function as indicated above (Level 3 again), (4) the selection of one individual complex organism from all those that may exist in this "world", including some "moment of awareness" for it (giving rise to an approximate localization of this observer in space and time: a subjective "here-and-now" – thus including Level 1), and (5) the selection of one of his/her/its "versions" that must have been created by further Everett branching based on the decoherence of matter variables according to Sect. 4 (Level 3). Therefore, any conceivable subjective observer who is part of the universe represents an extreme "individualization" (multiple localization) *in* the real quantum universe, and hence *of* his observed "world" (his frog's perspective). This individualization seems to be required in order to allow for definite IISs (integrated-information systems) or IGUSs (information gaining and utilizing systems) or whatever they are called, as a physical basis for conscious beings, and, in this way, for an *observable* universe. New physical laws do not seem to be required for this purpose.

Each of these steps may create its own unpredictable initial conditions characterizing the further evolution of the resulting individual worlds. Most properties characterizing our observed one can thus not be derived from any physical theory; they have to be empirically determined as part of an answer to the question: *Where* do *we* happen to live in objective "configuration" space? This unpredictability, including that of certain "constants of nature", and complained about by some mathematical physicists and cosmologists, is by no means specific for a multiverse (as some critics argue). It would similarly apply to any kind of stochastic dynamics (such as in collapse theories), or whenever statistical fluctuations are relevant during the early cosmic evolution. Only step 4 can *not* be objectivized in the usual sense, namely with respect to different observers in the same quasi-classical world. Some of these steps may require an application of the weak anthropic principle in some sense (although I would not recommend to *rely* on it for the future by playing "Russian quantum roulette"!). However, none of the observed individual outcomes should have *exceptionally* small probability weight – still a strong condition, which would suffice to explain frequencies of measurement results according to Born's rule (Sect. 4). Entropy may *decrease* during most of these steps (depending on its precise definition).[6,9,26]

Let me add for completeness that Tegmark's Level 1 and 2 multiverses are classical concepts, and thus unrelated to Everett's branchings, as they merely refer to separate regions in conventional space rather than in "configuration" space. It appears somewhat pretentious to speak of "parallel worlds" or a "multiverse" in this case; these names were originally invented for Everett branches (separate wave packets in configuration space), and are here simply misused. The reason may be that many cosmologists had never accepted the role of entangled quantum superpositions as part of reality, and therefore prefer to tacitly replace them by statistical correlations characterizing a collapse mechanism. In this case, different outcomes could be realized only at different locations in a sufficiently large three-dimensional space, while different Everett "worlds" exist even for a finite (closed) universe that represents a traditional big bang. (Even different kinds and sizes of universes may formally exist in one superposition, though, if the superposition principle is universally valid.)

While landscapes of regions with different properties would at least be conceivable in a spatially unbounded or very large universe without making use of Everett (similar to locally varying order parameters resulting from symmetry breaking phase transitions[18]), almost *identical* local situations occurring by chance somewhere in an infinite quasi-homogeneous world (Level 1) may be regarded as something between trivial (entirely irrelevant for us) and ill-defined. Although the double exponentials which are needed to describe the expected distances to such Doppelgängers can easily be *formulated*, an extrapolation of local properties (such as an approximately flat quasi-classical space) from the observable universe with its size of 10^{10} *ly* to something like $10^{10^{100}}$ *ly* appears at least risky. Statistical estimates of probabilities would in any case apply only to chance fluctuations (such as "Boltzmann brains"), but not to situations resulting from evolution. Their probabilities, calculated by means of some *physical* (that is, additive) entropy, would neglect the existence of "consistent documents" (often regarded as an "overdetermination of the past" – see Footnote ** above and Sect. 3.5 of the first Ref. 9), while *unstructured* initial conditions (such as the initial homogeneity of a gravitating universe) represents even lower entropy values – in spite of their "plausibility".

The role of Tegmark's (as yet unmentioned) Level 4 universes is even entirely questionable, since mathematics, while providing extremely useful conceptual tools for physics because of its analytical (tautological) nature and, therefore, the undeniable *formal truth* of its theorems, cannot by itself warrant the *applicability* of specific formal concepts to the empirical world. Only if, and insofar as, such kinematical concepts have been empirically verified to be consistently applicable in a certain context, can we consider them as candidates for a description of "reality". (This seems to be a point that many mathematicians working in theoretical physics and cosmology have problems to understand, since they are used to define their concepts just for convenience.) Different mathematical frameworks can therefore not be regarded as indicating the existence of corresponding different physical "worlds" or different parts of one world. While Everett's "many worlds" (just as all scientific cosmology) result from hypothetical extrapolation of the observed world by means of empirical laws, there are no reasons supporting the physical existence of Level 4 worlds. The *mathematical* concept of "existence", for example, means no more than the absence of logical inconsistencies, that is, a necessary (hence important[27]) but not a sufficient condition for being "realized" in Nature.

7. Conclusions

These remarks about quantum gravity and quantum cosmology may bring the strange story of particles and waves in principle to a (preliminary) end. While the particle concept has been recognized as a mere illusion, the observed wave aspects of microscopic objects can be consistently understood only as part of a universal wave function in a very high dimensional (if not infinite-dimensional) "configuration" space. Only if one insists that reality has to be defined in space and time, then, trivially, there can be no reality at all any more. This seems indeed to be the most common prejudice that prevents physicists from understanding quantum theory. The observable quantum world (our frog's perspective, that is, the individual relative state with respect to a "single mind" in the global superposition) is described by no more than a tiny component of this global wave function. The latter, representing the bird's view, may be regarded as the true "hidden" reality behind all phenomena, as it is only required for dynamical consistency. For example, the Wheeler-DeWitt wave function seems to be meaningful essentially only from this bird's perspective, since it describes a superposition of many different spacetimes (which is immediately decohered by matter).

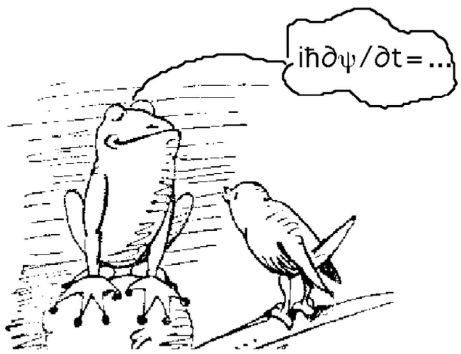

Fig. 2: A frog's bird's perspective

Matrix mechanics with its formal concept of "observables" thus turns out to be only an effective local probabilistic description in terms of not consistently applicable (hence "complementary") particle or other traditional concepts, which may in certain situations approximately apply to the observed world (our branch). Many physicists are busy constructing absurdities, paradoxes, or no-go theorems in terms of such classical variables in order to demonstrate the "weirdness" of quantum theory. Even Alice and Bob are classical concepts that have quantum mechanically to be justified by means of decoherence, caused locally by an uncontrollable environment! "Quantum Bayesianism", presently much *en vogue*, does no more than replace the whole physical world by a black box, representing an abstract concept of "information" about inconsistent classical concepts, and assumed to be available to some vaguely defined "agents" rather than to observers who may be consistent parts of the physical world to be described. In contrast to Many Worlds, for example, such a "non-theory" can never be falsified (it is "not even wrong").

While concepts like particles and spatial fields remain important for our every-day life, including that in physics laboratories, their limited validity must deeply affect a consistent world model (cosmology, in particular). It is always amazing to observe how the love affair of mathematical physicists and general relativists with their various classical field models often prevents them from consistently accepting, or even from sufficiently understanding, elementary quantum mechanics with all its surprising but important consequences. Some of them are even trying to explain the well understood quantum nonlocality by means of speculative "worm holes" in space – apparently because they are still believing in local reality. We have to accept, however, that the precise structure of a local Hilbert space *basis*, which is often assumed to be given by the configuration space of some fundamental fields, remains elusive. Because of the unavoidable entanglement of all variables, one cannot expect the *effective* quantum fields, which describe apparent "elementary particles", to be related to these elusive fundamental variables in a simple way. This conclusion puts in doubt much of the traditional approach to QFT, which is based on concepts of renormalization and "dressing".

There are indeed excellent arguments why even emergent ("effective") or quasi-classical fields may be mathematically elegant – thus giving rise to the impression of their fundamental nature. Novel mathematical concepts might nonetheless be required for finding the elusive ultimate theory, but their applicability to physics has to be demonstrated empirically, and can thus never be proved to be *exactly* valid. This may severely limit the physical value of many "abstract" (non-intuitive) mathematical theorems. Just think of Einstein's words "*Insofern sich die Sätze der Mathematik auf die Wirklichkeit beziehen, sind sie nicht sicher, und insofern sie sicher sind, beziehen sie sich nicht auf die Wirklichkeit*", or Feynman's remark regarding early attempts to quantize gravity:[3] "Don't be so rigorous or you will not succeed." Fundamental physical laws and concepts have so far mostly turned out to be mathematically relatively simple, while their applications may be highly complex. This fact may explain why mathematicians have dominated theoretical physics preferentially *after* completion of a new fundamental theory (such as Newton's and even more so Einstein's), or at times of stagnation, when mere reformulations or unconfirmed formal speculations (such as strings at the time of this writing) are often celebrated as new physics.

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(Refs. 28 and 29 appear in Footnote ** on page 26)